MA161 Quiz 23 Solutions

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Problem 23.1. The velocity of a particle at various times is given in the following table:

time t in s	0	0.25	0.5	0.75	1
velocity $v(t)$ in m/s	3	2	3	4	5

Estimate the distance traveled by the particle from time $t = 0$ to $t = 1$ with a **right** Riemann sum.

Solution. First, from the table we can deduce the information: $\Delta t = 0.25$, $t_0 = 0$, $t_1 = 0.25$, $t_2 = 0.5$, $t_3 = 0.75$, $t_4 = 1$, and $v(t_0) = 3$, $v(t_1) = 2$, $v(t_2) = 3$, $v(t_3) = 4$, $v(t_4) = 5$. Therefore, using right Riemann sums, the approximate distance traveled is

$$\text{dist.} \approx \sum_{i=1}^{n} v(t_i)\Delta t = 0.25(2+3+4+5) = \frac{14}{4} = \boxed{\frac{7}{2} = 3.5}.$$

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Problem 23.2. Estimate

$$\int_0^\pi \sin(x)\,dx$$

using a **right** Riemann sum with $n = 4$ rectangles.

Solution. First, we note that $\Delta x = (\pi - 0)/4 = \pi/4$. Then, $x_0 = 0$, $x_1 = \pi/4$, $x_2 = \pi/2$, $x_3 = 3\pi/4$, $x_4 = \pi$. Therefore, the approximation of the integral using right Riemann sum with $n = 4$ is

$$\int_0^\pi \sin(x)\,dx \approx \tfrac{\pi}{4}(\sin(\pi/4) + \sin(\pi/2) + \sin(3\pi/4) + \sin(\pi)) = \boxed{\tfrac{\pi}{4}(1+\sqrt{2})}.$$

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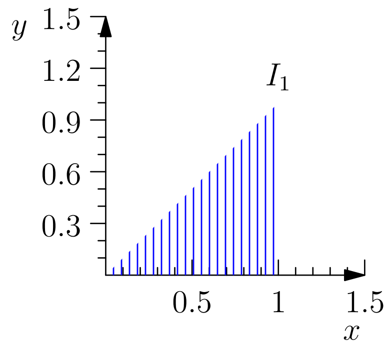
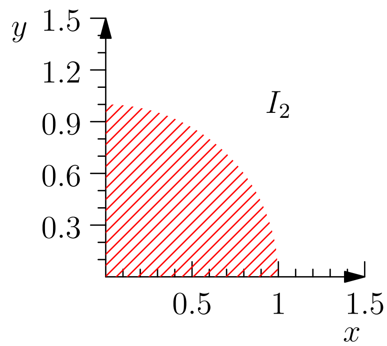
Problem 23.3. Find
$$\int_0^1 x - \sqrt{1-x^2}\, dx$$
by interpreting it in terms of areas.

Solution. To do this problem you have to first separate the integrals like so
$$\int_0^1 x - \sqrt{1-x^2}\, dx = \underbrace{\int_0^1 x\, dx}_{I_1} - \underbrace{\int_0^1 \sqrt{1-x^2}\, dx}_{I_2}.$$
Then the integral we are after is the difference $I_1 - I_2$. To find I_1 and I_2 we just have to draw the graphs of x and $\sqrt{1-x^2}$ respectively. They are: for x



and for $\sqrt{1-x^2}$



It is clear that in the first image, the area under the curve $I_1 = 1/2$ (from the area of a triangle which is half of the base times the height) and $I_2 = \pi/4$ since, as the image shows, I_2 is a quarter the area of a circle with radius $r = 1$. Therefore,
$$\int_0^1 x - \sqrt{1-x^2}\, dx = I_1 - I_2 = \boxed{\frac{1}{2} - \frac{\pi}{4}}.$$
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