MA161 Quiz 3

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Problem 3.1. Find the inverse of the following functions:

(a) $f(x) = e^{4x-9}$

(b) $g(x) = \frac{1}{4}\ln(x^2 - 2x + 1)$

Solution. Using the method you learned in class (that is, the one where you replace xs by ys and you solving in terms of x) we can easily find the inverses of both (a) and (b), as follows.

For part (a),

$$\begin{aligned} x &= e^{4y-9} \\ \ln(x) &= 4y - 9 \\ \frac{\ln(x) + 9}{4} &= y, \end{aligned}$$

so the inverse is $f^{-1}(x) = \frac{1}{4}(\ln(x) + 9)$.

For part (b),

$$\begin{aligned} x &= \frac{1}{4}\ln(y^2 - 2y + 1 \\ &= \frac{1}{4}\ln(y-1)^2 \\ &= \frac{1}{2}\ln(y-1) \\ 2x &= \ln(y-1) \\ e^{2x} &= y - 1 \\ e^{2x} + 1 &= y, \end{aligned}$$

so the inverse is $g^{-1}(x) = e^{2x} + 1$. ☺

Problem 3.2. Find the **exact** values of the following expressions:

(a) $\log_\pi(\pi^{-e})$

(b) $\log_{\sqrt{5}}(5)$.

(c) $\log_3(27^{1/2})$

Solution. All of these problems follow from the definition of logarithm. Do you remember what that was? The logarithm (base b) of a number x is that number y which solves the equation $b^y = x$. This can be a little bit hard to digest at first so an example is due here, and we will use this problem for that.

For part (a), $\log_\pi(\pi^{-e})$, in terms of what I said above, is saying

"Which x solves $\pi^x = \pi^{-e}$?"

It is easy to see that $x = -e$.

For part (b), again we ask

"Which x solves $(\sqrt{5})^x = 5$?"

Clearly if you square $\sqrt{5}$ you get 5, so $x = 2$.

For part (c), you know the drill. Write 27 as 3^3 and use the above logic, i.e., $3^x = (3^3)^{1/2} = 3^{3/2}$ when $x = 3/2$.

You will eventually get the hang of this. ☺

Problem 3.3. Sketch the graph of the following function

$$y = \ln(e^{x^2-1}) - \ln(e^{x+1}).$$

Solution. Depending on how much attention you paid to the recitation, this problem was either extremely easy, or you had no clue how to do it.

The following identities were useful

(i) $\ln(x) - \ln(y) = \ln(x/y)$

(ii) $e^x/e^y = e^{x-y}$.

Therefore,

$$
\begin{aligned}
y &= \ln(e^{x^2-1}) - \ln(e^{x+1}) \\
&= \ln\left(\frac{e^{x^2-1}}{e^{x+1}}\right) \\
&= \ln\left(e^{x^2-1-(x+1)}\right) \\
&= \ln\left(e^{x^2-x-2}\right) \\
&= x^2 - x - 2 \\
&= (x-2)(x+1);
\end{aligned}
$$

writing it as a product really helps you figure out what this quadratic looks like. All you need to do now is plot a couple of points. Plug in zero to this and you get the point of symmetry of the parabola which is $(0, -2)$, and then $(-1, 0)$, and $(2, 0)$ as the x-intercepts will finish up the picture. Here is an example ☺