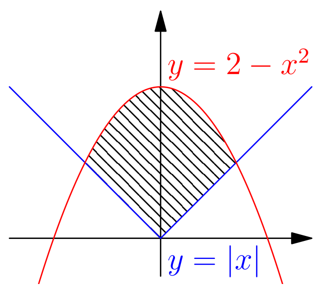
MA 162 Quiz 2
June 18, 2019

You have **15 minutes** to complete this quiz. Each correct answer will award you **five points**. Show your work **neatly** and you will receive **two** to **three points** depending on your level of correctness.

Problem 2.1. Find the area bounded by the curves $y = 2 - x^2$ and $y = |x|$ (*Hint:* Use symmetry to simplify your calculation.)

(A) 10/3 (B) 7/6 (C) 13/3 (D) 7/3 (E) 13/6

Solution. We begin by sketching the region in question:



From the image above, it is somewhat easy to see that the region to the left of the y-axis mirrors the region to the right so it is enough to find the area A underneath the right half. We do this now:

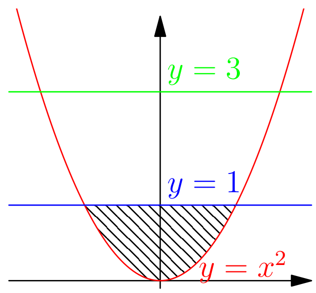
$$
\begin{aligned}
A &= \int_0^1 2 - x^2 - x\,dx \\
&= 2 - \frac{x^3}{3} - \frac{x^2}{2}\Big|_0^1 \\
&= 2 - \frac{1}{3} - \frac{1}{2} \\
&= \frac{12 - 2 - 3}{6} \\
&= \frac{7}{6}
\end{aligned}
$$

so the total area is $2A = \underline{7/3}$. ◇

Problem 2.2. Use cylindrical shells to find a formula for the volume of the region bounded by $y = x^2$ and $y = 1$ is revolved about the line $y = 3$.

(A) $\int_{-1}^{1} 2\pi(1-y)\sqrt{y}\,dy$ (B) $\int_0^1 2\pi(3-y)\sqrt{y}\,dy$ (C) $\int_0^1 4\pi(3-y)\sqrt{y}\,dy$
(D) $\int_0^1 2\pi(3-x)x^2\,dx$ (E) $\int_{-1}^{1} \pi(1-x^2)\,dx$.

Solution. We begin by sketching the region in question:



First note that by symmetry we need only consider the volume V the solid lying in the first quadrant, then the total volume will be $2V$. Having made this observation, by the shell method:

$$V = \int_0^1 2\pi(3-y)\sqrt{y}\,dy.$$

So the total volume is $2V = \underline{\int_0^1 4\pi(3-y)\sqrt{y}\,dy}$.

If you wrote down

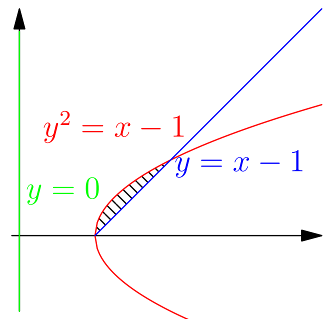
$$\int_0^1 2\pi(3-y)\sqrt{y}\,dy + \int_{-1}^{0} 2\pi(3-y)(-\sqrt{y})\,dy,$$

this will also be counted as correct. ◇

Problem 2.3. Find the volume of the solid that results from rotating the area between the curves $y^2 = x - 1$ and $y = x - 1$ about the y-axis.
(*Hint:* Use the washer method with cross sections in terms of y.)

(A) $3\pi/7$ (B) $7\pi/15$ (C) $\pi/6$ (D) $\pi/3$ (E) $\pi/10$

Solution. We begin by sketching the region in question:



By the washer method, the volume V of the solid is

$$\begin{aligned}
V &= \int_0^1 \pi((y+1)^2 - (y^2+1)^2)\, dy \\
&= \pi \int_0^1 (y^2 + 2y + 1 - y^4 - 2y^2 - 1)\, dy \\
&= \pi \int_0^1 (-y^4 - y^2 + 2y)\, dy \\
&= \pi \left(-\frac{y^5}{5} - \frac{y^3}{3} + y^2 \right)\Bigg|_0^1 \\
&= \pi \left(\frac{-3 - 5 + 15}{15} \right) \\
&= \frac{7\pi}{15}.
\end{aligned}$$

$\diamond$