MA 162 Quiz 3

June 20, 2019

You have **15 minutes** to complete this quiz. Each correct answer will award you **five points**. Show your work **neatly** and you will receive **two** to **three points** depending on your level of correctness.

Problem 3.1. Which of the following integrals give the length of the curve $y = \sin\sqrt{x}$ on the interval $a \le x \le b$?

(A) $\int_a^b \sqrt{x + \cos^2\sqrt{x}}\,dx$ (B) $\int_a^b \sqrt{1 + \cos^2\sqrt{x}}\,dx$ (C) $\int_a^b \sqrt{\sin^2\sqrt{x} + \frac{1}{4x}\cos^2\sqrt{x}}\,dx$

(D) $\int_a^b \sqrt{1 + \frac{1}{4x}\cos^2\sqrt{x}}\,dx$ (E) $\int_a^b \sqrt{\frac{1+\cos^2\sqrt{x}}{4x}}\,dx$

Solution. By the arclength formula:

$$\int_a^b \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\cos\sqrt{x}\right)^2}\,dx = \underline{\int_a^b \sqrt{1 + \frac{1}{4x}\cos^2\sqrt{x}}\,dx}.$$

◇

Problem 3.2. Which of the following integrals gives the surface area of the surface of revolution formed by revolving $y = 1/x$, $a \le x \le b$ about the x-axis?

(A) $\int_a^b \frac{2\pi}{x}\,dx$ (B) $\int_a^b \frac{2\pi}{x^2}\,dx$ (C) $\int_a^b 2\pi\frac{\sqrt{1+x^4}}{x^3}\,dx$ (D) $\int_a^b 2\pi\left(1 + \frac{1}{x^2}\right)\,dx$

(E) $\int_a^b 2\pi\sqrt{1 + \frac{1}{x^4}}\,dx$

Solution. By the surface area formula (for surfaces of revolution about the

x-axis):

$$\begin{aligned}\int_a^b \frac{2\pi}{x}\sqrt{1+\left(\frac{1}{x}\right)^2}\,dx &= \int_a^b \frac{2\pi}{x}\sqrt{1+\frac{1}{x^4}}\,dx\\ &= \int_a^b 2\pi\sqrt{\frac{1}{x^2}+\frac{1}{x^6}}\,dx\\ &= \int_a^b 2\pi\sqrt{\frac{x^4+1}{x^6}}\,dx\\ &= \underline{\int_a^b 2\pi\frac{\sqrt{x^4+1}}{x^3}\,dx}.\end{aligned}$$

◇

Problem 3.3. A spring has a natural length of 5 m. If a 25 N force is required to keep it stretch to a length of 10 m, how much work (in Joules) is required to stretch it from 5 m to 6 m?

(A) 55/2 (B) 55/4 (C) 5/4 (D) 5/2 (E) 5

Solution. Note that the natural length is the equilibrium position so the spring is stretch by 5 m from its equilibrium position. By Hooke's law, $F = kx$ so $k = F/x = 25/5 = 5$ N·m. Thus, the work required to stretch this from 5 m to 6 m is (i.e. 1 m from equilibrium position) is

$$W = \frac{1}{2}kx^2 = \frac{5\cdot 1^2}{2} = \underline{5/2\ \text{J}}.$$

◇