MA 162 Quiz 4
June 27, 2019

You have **15 minutes** to complete this quiz. Each correct answer will award you **five points**. Show your work **neatly** and you will receive **two** to **three points** depending on your level of correctness.

Problem 4.1. Evaluate the integral

$$\int_0^1 xe^{2x}\,dx.$$

(*Hint:* Use integration by parts.)

(A) $1+2e^2$ (B) $\dfrac{1+3e^2}{2}$ (C) $\dfrac{1+e^2}{4}$ (D) $1+3e^2$ (E) $\dfrac{1+2e^2}{4}$

Solution. Using integration by parts with $u=x$ and $dv=e^{2x}$,

$$\begin{aligned}\int_0^1 xe^{2x}\,dx &= \frac{1}{2}xe^{2x}\Big|_0^1 - \frac{1}{2}\int_0^1 e^{2x}\,dx\\ &= \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}\Big|_0^1\\ &= \underline{\frac{1+e^2}{4}}.\end{aligned}$$

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Problem 4.2. Evaluate the integral

$$\int_0^{\pi/2} \cos^4 x\,dx.$$

(*Hint:* Use the identity $2\cos^2(x) = 1+\cos(2x)$.)

(A) $\dfrac{\pi}{6}$ (B) $\dfrac{2\pi}{9}$ (C) $\dfrac{3\pi}{16}$ (D) $\dfrac{\pi}{5}$ (E) $\dfrac{5\pi}{8}$

Solution. By applying the identity, we get

$$\begin{aligned}\int_0^{\pi/2}\cos^4 x\,dx &= \int_0^{\pi/2}\left(\frac{1+\cos(2x)}{2}\right)^2 dx\\ &= \frac{1}{4}\int_0^{\pi/2} 1+2\cos(2x)+\cos^2(2x)\,dx\end{aligned}$$

applying the identity again on $\cos^2(2x)$, the above becomes

$$= \frac{1}{4}\int_0^{\pi/2}\left(1 + 2\cos(2x) + \frac{1+\cos(4x)}{2}\right) dx$$
$$= \frac{1}{4}\left(\frac{3}{2}x + \sin(2x) + \frac{1}{8}\sin(4x)\right)\Big|_0^1$$
$$= \frac{3\pi}{16}.$$

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Problem 4.3. Evaluate the integral

$$\int_0^2 \frac{1}{(x^2+4)^{3/2}}\, dx$$

(*Hint:* Use a trigonometric substitution.)

(A) $\frac{\sqrt{2}}{8}$ (B) $\frac{\sqrt{2}}{4}$ (C) $\frac{\sqrt{2}}{2}$ (D) $\frac{\sqrt{2}}{32}$ (E) $\frac{\sqrt{2}}{16}$

Solution. There are two triangles corresponding to this problem, one whose base is x and the other whose base is 2.

For with the base equal to x, we can make the following substitution: $\tan\theta = 2/x$ so $x = 2\cot\theta$ and $dx = -2\csc\theta$ and $\sqrt{x^2+4} = 2\csc\theta$. Then

$$\int_0^2 \frac{1}{(x^2+4)^{3/2}}\, dx = \int_{\pi/2}^{\pi/4} \frac{1}{2^3\csc^3\theta}(-2\csc^2\theta)\, d\theta$$
$$= \frac{1}{4}\int_{\pi/2}^{\pi/4} -\sin\theta\, d\theta$$
$$= \frac{1}{4}\cos\theta\Big|_{\pi/2}^{\pi/4}$$
$$= \frac{1}{4}\frac{\sqrt{2}}{2} - \frac{1}{4}\cdot 0$$
$$= \frac{\sqrt{2}}{8}.$$

Doing it the other way is equally straightforward: From the triangle, $\tan\theta = x/2$ so we make the substitution $x = 2\tan\theta$ and $dx = 2\sec^2\theta\, d\theta$, and

$\sqrt{x^2+4} = 2\sec\theta$. Thus,

$$\begin{aligned}
\int_0^2 \frac{1}{(x^2+4)^{3/2}}\,dx &= \int_0^{\pi/4} \frac{1}{2^3\sec^3\theta}(2\sec^2\theta)\,d\theta \\
&= \frac{1}{4}\int_0^{\pi/4} \cos\theta\,d\theta \\
&= \frac{1}{4}\sin\theta\Big|_0^{\pi/4} \\
&= \frac{1}{4}\left(\frac{\sqrt{2}}{2} - 0\right) \\
&= \frac{\sqrt{2}}{8}.
\end{aligned}$$

Note that the bounds change depending on the method we use, but that is to be expected. ◇