MA 162 Quiz 5
July 2, 2019

You have **15 minutes** to complete this quiz. Each correct answer will award you **five points**. Show your work **neatly** and you will receive **one** to **three points** depending on your level of correctness.

Problem 5.1. The partial fractions decomposition of

$$f(x) = \frac{x}{(x-1)(x-2)(x-3)}$$

is of the form

$$\frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}.$$

Find the value of B.

(A) -2 (B) $\frac{1}{2}$ (C) -1 (D) 3 (E) 0

Solution. By the cover-up method,

$$\frac{x}{(x-1)(x-3)} = \frac{A}{x-1}(x-2) + B + \frac{C}{x-3}(x-2)$$

plugging in $x = 2$, we get

$$\begin{aligned} \frac{2}{(2-1)(2-3)} &= \frac{A}{2-1}(2-2) + B + \frac{C}{2-3}(2-2) \\ \frac{2}{(1)(-1)} &= \frac{A}{1} \cdot 0 + B + \frac{C}{-1} \cdot 0 \\ -2 &= B. \end{aligned}$$

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Problem 5.2. If you expand $(2x+1)/(x^3+x)$ as a partial fraction, which expression below would you get?

(A) $\frac{1}{x} + \frac{-x+2}{x^2+1}$ (B) $\frac{-1}{x^2} + \frac{1}{x+1}$ (C) $\frac{2}{x} + \frac{1}{x^2+1}$

(D) $\frac{-1}{x} + \frac{x}{x^2+1}$ (E) $\frac{-2}{x} + \frac{1}{x^2+1}$

Solution. By factoring the denominator into $x(x^2+1)$ we know that the partial fraction decomposition has the form

$$\frac{A}{x} + \frac{Bx+C}{x^2+1}.$$

By the cover-up method, we may easily find the value of A

$$\begin{aligned}
\frac{2x+1}{x(x^2+1)} &= \frac{A}{x} + \frac{Bx+C}{x^2+1} \\
\frac{2x+1}{x^2+1} &= A + \frac{Bx+C}{x^2+1}x
\end{aligned}$$

plugging in $x = 0$, we get

$$\begin{aligned}
\frac{2 \cdot 0 + 1}{0^2+1} &= A + \frac{B \cdot 0 + C}{0^2+1} \cdot 0 \\
A &= 1.
\end{aligned}$$

This is enough to tell you what the right answer is. But for the sake of completion, let us find B and C, below

$$\begin{aligned}
\frac{2x+1}{x(x^2+1)} &= \frac{1}{x} + \frac{Bx+C}{x^2+1} \\
2x+1 &= x^2+1+(Bx+C)x \\
2x+1 &= (1+B)x^2 + Cx + 1.
\end{aligned}$$

Thus, $B = -1$ and $C = 2$. The partial fraction decomposition is therefore

$$\frac{1}{x} + \frac{-x+2}{x^2+1}.$$

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Problem 5.3. Evaluate the improper integral

$$\int_4^\infty \frac{1}{(x-2)(x-3)}\,dx.$$

(A) $\ln 3$ (B) $\ln(1/2)$ (C) $\ln 2$ (D) $3\ln 2$ (E) the integral diverges

Solution. Note that the denominator is $(x-2)(x-3)$ so the partial fraction decomposition of the integrand is of the form

$$\frac{A}{x-2} + \frac{B}{x-3}.$$

By the cover-up method,

$$\frac{1}{x-3} = A + \frac{B}{x-3}(x-2) \implies A = -1$$

and

$$\frac{1}{x-2} = \frac{A}{x-2}(x-3) + B \implies B = 1.$$

Thus,

$$\begin{aligned}
\int_4^\infty \frac{1}{(x-2)(x-3)}\,dx &= \int_4^\infty \frac{-1}{x-2} + \frac{1}{x-3}\,dx \\
&= -\ln|x-2| + \ln|x-3|\Big|_4^\infty \\
&= \ln\left|\frac{x-3}{x-2}\right|\Big|_4^\infty \\
&= \lim_{x\to\infty} \ln\left|\frac{x-3}{x-2}\right| - \ln\left|\frac{4-3}{4-2}\right| \\
&= \ln(1) - \ln\left(\tfrac{1}{2}\right) \\
&= -\ln\left(\tfrac{1}{2}\right) \\
&= \ln\left(\left(\tfrac{1}{2}\right)^{-1}\right) \\
&= \ln 2.
\end{aligned}$$

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