MA 162 Quiz 6
July 11, 2019

You have **15 minutes** to complete this quiz. Each correct answer will award you **five points**. Show your work **neatly** and you will receive **one** to **three points** depending on your level of correctness.

Problem 6.1. The series

$$\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{\sqrt{n}}$$

(A) diverges even though $\lim_{n\to\infty}(-1)^{n+1}/\sqrt{n} = 0$.
(B) does not converge absolutely, but converges conditionally.
(C) diverges because $\lim_{n\to\infty}(-1)^{n+1}/\sqrt{n} \neq 0$.
(D) converges absolutely.
(E) diverges because the terms alternate.

Solution. Notice that

$$\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0,$$

and

$$\frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}},$$

so by the alternating series test, the series converges.

However, it is not absolutely continuous. In particular, the series $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n}}$ diverges by direct comparison with the harmonic series $\sum_{n=2}^{\infty} \frac{1}{n}$ since

$$\frac{1}{\sqrt{n}} > \frac{1}{n}.$$

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Problem 6.2. What series should we use in the limit comparison test in order to determine whether the following series converges?

$$\sum_{n=1}^{\infty} \frac{2^n}{3^n - 1}$$

(A) $\sum_{n=1}^{\infty} 3^n$ (B) $\sum_{n=1}^{\infty} \frac{1}{3^n - 1}$ (C) $\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n$ (D) $\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n$ (E) $\sum_{n=1}^{\infty} \frac{1}{n}$

Solution. The terms in the series most resemble $\left(\frac{3}{2}\right)^n$. Therefore, it makes the most sense to compare with the series

$$\sum_{n=1}^{\infty} \left(\frac{3}{2}\right)^n.$$

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Problem 6.3. By the limit comparison test, which one of the following series diverges?

(A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{3/2}+1}$ (B) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2+8}$ (C) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+100}$

(D) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{n+1}\left(\frac{1}{2}\right)^n$ (E) $\displaystyle\sum_{n=1}^{\infty} 7\left(\frac{5}{6}\right)^n$

Solution. There is in fact only one series which diverges and that is

$$\sum_{n=1}^{\infty} \frac{n^2-1}{n^3+100}.$$

This can be determined by inspection, as taking its leading terms gives us a the harmonic series. After comparing with, say, the series

$$S = \sum_{n=1}^{\infty} \frac{n^2}{100n^3},$$

we see that this series must diverge because S diverges by the limit comparison test with respect to the harmonic series. ◇