MA 162 Quiz 7
July 16, 2019

You have **15 minutes** to complete this quiz. Each correct answer will award you **five points**. Show your work **neatly** and you will receive **one** to **three points** depending on your level of correctness.

Problem 7.1. Which of these series diverge by the quick test for divergence?

(A) $\sum_{n=1}^{\infty} \sin(1/n)$ (B) $\sum_{n=1}^{\infty} 1/(5-e^{-n})$ (C) $\sum_{n=1}^{\infty} \ln(n)/(n+7)$
(D) $\sum_{n=1}^{\infty} \ln(n)/n$ (E) none of these

Solution. Note that

$$\lim_{n\to\infty} \sin(1/n) = \sin(0) = 0,$$
$$\lim_{n\to\infty} \frac{1}{5-e^{-n}} = \frac{1}{5},$$
$$\lim_{n\to\infty} \frac{\ln(n)}{n+7} = 0,$$
$$\lim_{n\to\infty} \frac{\ln(n)}{n} = 0,$$

so $\sum_{n=1}^{\infty} 1/(5-e^{-n})$ although the other sequences need not converge (and in fact do not). ◇

Problem 7.2. Which of the following statements are true?

A. If $\lim_{n\to\infty} a_n = 0$, then $\sum_n a_n$ converges.
B. The ratio test can be used to determine whether $\sum_n 1/n^3$ converges.
C. If $\sum_n a_n$ is divergent, then $\sum_n |a_n|$ is divergent.

(A) C only (B) A only (C) A and B only (D) B and C only (E) none of them

Solution. A need not be true since $\frac{1}{7}n \to 0$, but $\sum_{n=1}^{\infty} 1/n = \infty$.

B is not true since, by the ratio test,

$$\lim_{n\to\infty} \frac{n^3}{(n+1)^3} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{-3} = 1,$$

which is inconclusive.

C is true since, by direct comparison, $a_n \leq |a_n|$ so if $\sum_{n=k}^{\infty} a_n$ diverges so does $\sum_{n=k}^{\infty} |a_n|$. ◇

Problem 7.3. Which of the following series converge?

I. $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{n!}$ II. $\displaystyle\sum_{n=1}^{\infty} \frac{\sin^2(n)}{n+n^{3/2}}$ III. $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{n^{2/3}}$

(A) I only (B) II only (C) I and II only (D) none of them
(E) all of them

Solution. I converges by the ratio test since

$$\lim_{n\to\infty} \frac{e^{n+1}}{(n+1)!}\frac{n!}{e^n} = \lim_{n\to\infty} \frac{e}{n+1} = 0.$$

II converges since

$$\frac{\sin^2(n)}{n+n^{3/2}} \leq \frac{1}{n+n^{3/2}} < \frac{1}{n^{3/2}}$$

and the last of these comes from the p-series $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ with $p = 3/2 > 1$.

III converges by the alternating series test since

$$\lim_{n\to\infty} \frac{1}{n^{2/3}} = 0,$$

and

$$\frac{1}{(n+1)^{2/3}} < \frac{1}{n^{2/3}}.$$

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