# MA 162 Quiz 9
## July 25, 2019

You have **15 minutes** to complete this quiz. Each correct answer will award you **five points**. Show your work **neatly** and you will receive **one** to **three points** depending on your level of correctness.

**Problem 9.1.** Find the interval of convergence for

$$\sum_{n=1}^{\infty} \frac{3^n}{n^n}(x-5)^n.$$

(A) $(-1/3, 1/3)$ (B) $(5-e/3, 5+e/3)$ (C) $(5-e/3, 5+e/3)$ (D) $(5-1/3, 5+1/3)$
(E) $(-\infty, \infty)$

*Solution.* By Hadamard's formula for the radius of convergence,

$$\frac{1}{R} = \lim_{n\to\infty} \left|\frac{3^n}{n^n}\right|^{1/n} = \lim_{n\to\infty} \frac{3}{n} = 0.$$

Therefore, $R = \infty$ so the interval of convergence must be $(-\infty, \infty)$. $\diamond$

**Problem 9.2.** The first three nonzero terms of the Maclaurin series of $f(x) = x/(1+x^3)$ are

(A) $x + x^4 + x^7$ (B) $x - x^4 + x^7$ (C) $1 - x^3 + x^6$ (D) $1 + x^3 + x^6$
(E) $-x - x^4 - x^7$

*Solution.* To find the Maclaurin series of $f(x) = x/(1+x^3)$ we need not find any of the derivatives since

$$f(x) = x\left(\frac{1}{1-(-x^3)}\right) = x\sum_{n=0}^{\infty}(-1)^n x^{3n} = \sum_{n=0}^{\infty}(-1)^n x^{3n+1}.$$

Expanding this out, we get

$$x - x^4 + x^7 - x^{10} + \cdots.$$

$\diamond$

**Problem 9.3.** Use the fact that

$$\frac{2x}{(1-x^2)^2} = \frac{d}{dx}\left(\frac{1}{1-x^2}\right)$$

to find a power series for $2x/(1-x^2)^2$.

(A) $\sum_{n=1}^{\infty} 2nx^{2n-1}$ (B) $\sum_{n=1}^{\infty}(-1)^n(2n+1)x^{2n+1}$ (C) $\sum_{n=1}^{\infty}(2n-1)x^{2n+1}$ (D) $\sum_{n=1}^{\infty} nx^{2n+1}$
(E) $\sum_{n=1}^{\infty} \frac{1}{n+1}x^{2n-1}$

*Solution.* The power series for $1/(1-x^2)$ is

$$\sum_{n=0}^{\infty} x^{2n}.$$

Thus, the power series for $2x/(1-x^2)^2$ is

$$\frac{d}{dx}\sum_{n=0}^{\infty} x^{2n} = \sum_{n=1}^{\infty} 2nx^{2n-1}.$$

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