MA 261 Quiz 10
November 13, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 10.1. Evaluate the line integral

$$\oint_C xy\,dx + x^2y^3\,dy,$$

where C is the boundary of the triangle with vertices at $(0,0)$, $(1,0)$, and $(1,4)$, oriented counterclockwise. *Hint*: Use Green's Theorem.

(A) 24
(B) 22
(C) 20
(D) 2π
(E) I don't know

Solution. By Green's Theorem, we need only find P and Q and evaluate the area integral over the function from the theorem. That is, with $P = xy$ and $Q = x^2y^3$, the theorem yields

$$\begin{aligned}\oint_C xy\,dx + x^2y^3\,dy &= \iint_D 2xy^3 - x\,dA \\ &= \int_0^1 \int_0^{4x} 2xy^3 - x\,dydx \\ &= \int_0^1 2\cdot 4^3x^5 - 4x^2\,dx \\ &= \frac{4^3-4}{3} \\ &= \frac{4(4^2-1)}{3} \\ &= \frac{4\cdot 15}{3} \\ &= 20.\end{aligned}$$

Therefore, the correct answer was (C). ◇

Problem 10.2. Compute the line integral

$$\oint_C x^2\,dy,$$

where C is the boundary of the rectangle with vertices $(0,0)$, $(2,0)$, $(2,3)$, and $(0,3)$, oriented counterclockwise. *Hint*: Use Green's Theorem.
(A) 4
(B) 8
(C) 12
(D) 16
(E) I don't know

Solution. By Green's Theorem, we need only find P and Q and compute the area integral. One quickly sees that $Q = x^2$ and so, by Green's Theorem,

$$\begin{aligned}\oint_C x^2\,dy &= \iint_D 2x\,dA \\ &= \int_0^2 \int_0^3 2x\,dy\,dx \\ &= \int_0^2 6x\,dx \\ &= 3x^2\Big|_0^2 \\ &= 12.\end{aligned}$$

Therefore, the correct answer was (C). ◇

Problem 10.3. If $f(x,y,z) = x^2yz - xy^2 + 2xz^2$ then $\nabla\cdot(\nabla f)$ at $(1,1,1)$ is equal to:
(A) 1
(B) 2
(C) 3
(D) 4
(E) I don't know

Solution. If you remember the equations for the gradient of a function, and the divergence of a vector field, this should come easily. The calculation goes as follows,

$$\begin{aligned}
\nabla f &= \nabla(x^2yz - xy^2 + 2xz^2) \\
&= \langle\, 2xyz - y^2 + 2z^2, x^2z - 2xy, x^2y + 4xz \,\rangle \\
\nabla \cdot \nabla f &= \nabla \cdot \langle\, 2xyz - y^2 + 2z^2, x^2z - 2xy, x^2y + 4xz \,\rangle \\
&= 2yz - 2x + 4z.
\end{aligned}$$

Plugging in the point $(1, 1, 1)$ into this last expression, we g $\nabla \cdot \nabla f(1, 1, 1) = 4$. Therefore, the correct answer was (D) ◇