MA 261 Quiz 11
November 27, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 11.1. Find the surface area of the parametric surface $\mathbf{r}(u,v) = \langle\, u+v, v, u \,\rangle$ with $0 \le u \le \pi$, $0 \le v \le \sqrt{3}$.

(A) 4π
(B) 2π
(C) $\pi\sqrt{3}$
(D) 3π
(E) I don't know

Solution. Let S be the surface parameterized by $\mathbf{r}(u,v)$ as defined in the statement of the problem, and D the domain of $\mathbf{r}(u,v)$. By the surface area formula,

$$A(S) = \iint_D |\mathbf{r}_u(u,v) \times \mathbf{r}_v(u,v)|\, dA. \tag{SA}$$

First, we need to find $\mathbf{r}_u$ and $\mathbf{r}_v$, and their cross-product. These are

$$\begin{aligned} \mathbf{r}_u(u,v) &= \langle\, 1, 0, 1 \,\rangle, \\ \mathbf{r}_v(u,v) &= \langle\, 1, 1, 0 \,\rangle, \\ \mathbf{r}_u \times \mathbf{r}_v(u,v) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix} \\ &= \langle\, -1, 1, 1 \,\rangle. \end{aligned}$$

Going back to (SA),

$$\begin{aligned} A(S) &= \int_0^{\pi} \int_0^{\sqrt{3}} |\langle\, -1, 1, 1 \,\rangle|\, dv du \\ &= \int_0^{\pi} \int_0^{\sqrt{3}} \sqrt{3}\, dv du \\ &= \int_0^{\pi} 3\, du \\ &= 3\pi. \end{aligned}$$

Therefore, the correct answer was (D). ◇

Problem 11.2. Let S be the parametric surface $\mathbf{r}(u,v) = \langle\, v\cos u, v\sin u, 2v^2 \,\rangle$ with $0 \le u \le 2$, $0 \le v \le 2$. Then S is part of a

(A) paraboloid
(B) cone
(C) cylinder
(D) ellipsoid
(E) I don't know

Solution. As mentioned in recitation, sometimes it is better to try to figure out whether we can write the parameterized surface as an equation, i.e., a relation of the form $z = f(x,y)$ or, in this case, $z = f(r,\theta)$. First, note that, since $x = v\cos u$, and $y = v\sin u$, so

$$r^2 = x^2 + y^2 = v^2\cos^2 u + v^2\sin^2 u = v^2.$$

Therefore, $z = 2r^2$. If you remember the table of quartic surfaces, this is a paraboloid.

Thus, the correct answer was (A). ◇

Problem 11.3. If the surface S is parameterized by $\mathbf{r}(u,v) = \langle\, u, v, uv^2 \,\rangle$, then the equation of the plane tangent to S at $(1,2,4)$ is

(A) $4x + 4y + z = 16$
(B) $4x + 4y - z = 8$
(C) $x + 2y - 2z = -3$
(D) $x + 2y + 2z = 13$
(E) I don't know

Solution. As shown in recitation, the equation for the tangent plane to a parameterized surface at a point is given by the formula

$$0 = \langle\, \mathbf{r}_u \times \mathbf{r}_v(u_0,v_0) \,\rangle \cdot \langle\, x - x_0, y - y_0, z - z_0 \,\rangle, \tag{TP}$$

where (x_0,y_0,z_0) is the point in question and (u_0,v_0) is the point in the domain of $\mathbf{r}$ where $r(u_0,v_0) = (x_0,y_0,z_0)$.

To use this formula, we first need to find (u_0,v_0). This is rather straightforward since a moment's glance tells us that, by the way $\mathbf{r}$ is defined, $u_0 = 1$ and $v_0 = 2$.

Thus,

$$\begin{aligned}
\mathbf{r}_u(u,v) &= \langle\, 1, 0, v^2 \,\rangle, \\
\mathbf{r}_v(u,v) &= \langle\, 0, 1, 2uv \,\rangle, \\
\mathbf{r}_u \times \mathbf{r}_v(u,v) &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & v^2 \\ 0 & 1 & 2uv \end{vmatrix} \\
&= \langle\, -v^2, -2uv, 1 \,\rangle.
\end{aligned}$$

Plugging in $(1,2)$ into the last equation, gives us the normal to the plane , i.e., $\mathbf{r}_u \times \mathbf{r}_v(1,2) = \langle\, -4, -4, 1 \,\rangle$. Hence, by (TP), the tangent plane is

$$\begin{aligned}
0 &= \langle\, -4, -4, 1 \,\rangle \cdot \langle\, x-1, y-2, z-4 \,\rangle \\
&= -4x - 4y + z + 4 + 8 - 4 \\
-8 &= -4x - 4y + z \\
8 &= 4x + 4y - z.
\end{aligned}$$

Therefore, the correct answer was (B). ◇