MA 261 QUIZ 3
SEPTEMBER 11, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive two points for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **PUID and section number**.

Problem 3.1. A particle is moving with acceleration

$$\mathbf{a}(t) = \langle 6, 6t, 0 \rangle.$$

If at time $t = 0$, $\mathbf{v}(0) = \langle 0, 0, 1 \rangle$, what is $\mathbf{v}(t)$?

(A) $\langle 6t, 3t, 1 \rangle$
(B) $\langle 6t, 3t^2, 0 \rangle$
(C) $\langle 6t, 3t^2, -1 \rangle$
(D) $\langle 6t, 3t^2, 1 \rangle$
(E) I don't know.

Solution. This problem is quite easily solved by integrating each component individually. If we do, we see that $\mathbf{v}(t) = \langle 6t, 3t^2, 0 \rangle + C$. We are told that at $t = 0$, $\mathbf{v}(0) = \langle 0, 0, 1 \rangle$. Therefore, $\mathbf{v}(t) = \langle 6t, 3t^2, 1 \rangle$, which is answer choice (D). $\diamond$

Problem 3.2. The position of a particle is given by $\mathbf{r}(t) = \langle 8\cos t, 3t, 8\sin t \rangle$. What is the speed $|\mathbf{v}|$ at $t = \pi$?

(A) $\sqrt{73}$
(B) 3
(C) $\sqrt{4 + 9\pi^2}/\pi$
(D) $\sqrt{4 + 9\pi^2}$
(E) I don't know.

Solution. Taking the derivative of each coordinate individually, we see that $\mathbf{v}(t) = \langle -8\sin t, 3, 8\cos t \rangle$. Therefore, the speed of $\mathbf{r}$ is

$$\begin{aligned} |\mathbf{v}(t)| &= \sqrt{64(\sin t)^2 + 9 + 64(\cos t)^2} \\ &= \sqrt{64 + 9} \\ &= \sqrt{73}. \end{aligned}$$

Since $|\mathbf{v}|$ is independent of t we are done. The correct answer choice is (A). $\diamond$

Problem 3.3 (Problem # 6 from Spring 2018). Find the length of the curve

$$\mathbf{r}(t) = \langle 4\sin t, 3t, -4\cos t\rangle, \quad 0 \le t \le 1/2$$

(A) $8\sinh^{-1}(3/8)/3$
(B) $8\sinh^{-1}(3/8)/3 + \sqrt{73}$
(C) $5/2$
(D) 5π
(E) I don't know.

Solution. To find the length of the curve $\mathbf{r}$ from $t = 0$ to $t = 1/2$, we must first find the derivative:

$$\mathbf{r}'(t) = \langle 4\cos t, 3, 4\sin t\rangle.$$

Then,

$$\int_0^{1/2} \sqrt{16(\cos t)^2 + 9 + 16(\sin t)^2}\, dt = \int_0^{1/2} \sqrt{25}\, dt = 5/2.$$

Therefore, the correct answer choice is (C). $\diamond$