MA 261 Quiz 4

September 18, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **PUID and section number**.

Problem 4.1. Determine (if it exists): $\lim_{(x,y)\to(0,0)} \dfrac{3x-2y}{\sqrt{x^2+y^2}}$.

(A) -2
(B) 0
(C) 3
(D) DNE
(E) I don't know

Solution. If the limit of $(3x-2y)/\sqrt{x^2+y^2}$ as $(x,y)\to 0$ exists, it should not matter how we approach $(0,0)$. Therefore, consider the following: Take the limit as we approach from the x-axis, that is, $(x,0)\to(0,0)$,

$$\lim_{(x,0)\to(0,0)} = \frac{3x}{\sqrt{x^2}} = 3.$$

On the other hand, if we take the limit as we approach from the y-axis, we get

$$\lim_{(0,y)\to(0,0)} \frac{-2y}{\sqrt{y^2}} = -2.$$

Since these two limits do not coincide, by definition, the limit at that point does not exist. Hence, the correct answer is (D). ◇

Problem 4.2. Let $f(x,y)=\ln(xy+x)$. Find f_{xy}.

(A) $1/(x+1)$
(B) 0
(C) y/x
(D) $xy/(xy+x)^2$
(E) I don't know

Solution. The easiest way to find the second partial derivative of f with respect to x and y is the following way:

$$\begin{aligned} f(x,y) &= \ln(xy+x) \\ &= \ln(((y+1)x) \\ &= \ln(y+1)+\ln(x) \end{aligned}$$

so

$$f_x(x, y) = \frac{1}{x}, \quad f_{xy}(x, y) = 0.$$

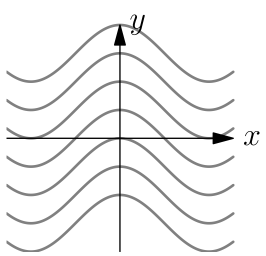
Hence, the correct answer choice was (B).

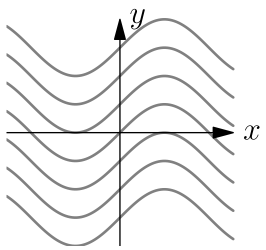
If you did not notice this, you could have still arrived at the right answer, but you would be doing it the hard way:

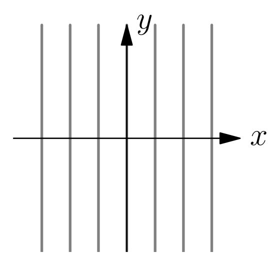
$$\begin{aligned} f_x(x, y) &= \frac{y+1}{xy+x}, \\ f_{xy}(x, y) &= \frac{(xy+x) - x(y+1)}{(xy+x)^2} \\ &= \frac{xy + x - xy - x}{(xy+x)^2} \\ &= 0. \end{aligned}$$

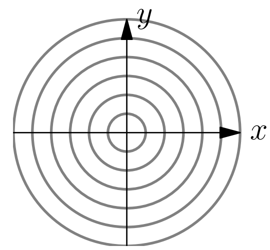
◇

Problem 4.3 (Fall 2017, # 5)**.** Suppose $z = f(x, y) = \cos x$. What is the correct contour map (level curves of f)?

(A) 

(B) 

(C) 

(D) 

(E) I don't know.

Solution. I believe I discussed the solution to this problem in class. It's always good to go back to old problems to remind ourselves how to do things. To find the contour map, choose a real number k such that the equation

$$k = \cos x$$

makes sense[1]. Then

$$x = \cos^{-1} k = \alpha + 2\pi k$$

where the right hand repeats periodically for k an integer (a whole positive or negative numbers). Therefore, the correct answer is (C). ◇

[1]Remember, the domain of cos is $[-1, 1]$ so as long as k is between -1 and 1, we are okay, but for this problem it is not important to precisely specify k because we are only concerned with an approximate picture and not the exact picture