MA 261 Quiz 5
September 25, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **PUID and section number**.

Problem 5.1. If

$$\begin{aligned} z &= \sin(xy), \\ x &= \pi t^2, \\ y &= h(t), \end{aligned}$$

with $h(1) = 1/3$ and $h'(1) = 2$, what is $dz/dt(1)$?

(A) $4\pi\sqrt{3}/3$
(B) $4\pi/3$
(C) $\pi + 1$
(D) $\sqrt{3}\pi + 1$
(E) I don't know

Solution. We will use the convenient notation $\dot{x}$ to mean dx/dt (this is often used in physics to distinguish a time derivative from a derivative with respect to some other quantity). Also note that $y = h$ so $\dot{y} = h'$. Carrying on, by the Chain Rule,

$$\begin{aligned} \dot{z}(t) &= \cos[x(1)y(1)](\dot{x}(1)y(1) + x(1)\dot{y}(1)) \\ &= \cos(\pi/3)(2\pi/3 + 2\pi) \\ &= \frac{4\pi}{3}. \end{aligned}$$

The correct answer was (B). ◇

Problem 5.2. Find the maximal rate of change in

$$f(x, y) = 5\ln(x + 2y) \quad \text{at } (1, 2).$$

(A) $\sqrt{5}$
(B) 3

(C) $\sqrt{125}/4$
(D) 5
(E) I don't know

Solution. The maximal rate of change happens in the direction of the gradient. First, let us find the gradient of f, which is

$$\nabla f = \left\langle \frac{5}{x+2y}, \frac{10}{x+2y} \right\rangle.$$

Now, as we said, the maximal rate of change of f at $(1,2)$ will happen in the direction $\mathbf{u}$ of ∇f, which is

$$\mathbf{u} = \frac{\langle 1,2\rangle}{\sqrt{5}}.$$

Therefore,

$$D_{\mathbf{u}} f(1,2) = \frac{1}{\sqrt{5}}\langle 1,2\rangle \cdot \langle 1,2\rangle = \sqrt{5} = |\nabla f|.$$

Note that this will always just be $|\nabla f(x_0, y_0)|$ (in this case $(x_0, y_0) = (1,2)$) so you do not need to do the unnecessary step of finding the direction of the gradient at a given point.

The correct answer was (A). ◇

Problem 5.3. Find the directional derivative of

$$f(x,y,z) = y^2 e^{x-z} \quad \text{at } (3,1,2)$$

in the direction $\langle 2,5,1\rangle$.
(A) $13e/\sqrt{30}$
(B) $13e$
(C) $11e$
(D) $11e/\sqrt{30}$
(E) I don't know

Solution. We first find the gradient of f at $(3,1,2)$, which is

$$\begin{aligned}\nabla f(3,1,2) &= \left\langle (1)^2 e^{3-2}, (2\cdot 1)e^{3-2}, -(1)^2 e^{3-2} \right\rangle \\ &= \langle e, 2e, -e\rangle.\end{aligned}$$

Now, to get the direction $\mathbf{u}$ of $\langle 2,5,1\rangle$, we normalize it like so

$$\mathbf{u} = \frac{1}{\sqrt{30}}\langle 2,5,1\rangle.$$

Therefore,

$$\begin{aligned} D_{\mathbf{u}} f &= \frac{1}{\sqrt{30}} \langle 2, 5, 1 \rangle \cdot \langle e, 2e, -e \rangle \\ &= \frac{11e}{\sqrt{30}}. \end{aligned}$$

Hence, the correct answer was (D). ◇