MA 261 Quiz 6

October 16, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 6.1. What is the value of the iterated integral

$$\iint_R 2 - x\,dA, \quad R = \{(x, y) \colon 0 \leq x \leq 2, 0 \leq y \leq 6\}?$$

(A) 6
(B) 8
(C) 10
(D) 12
(E) I don't know

Solution. This is a fairly straightforward computation. Just note that the bounds of your integral are $0 \leq x \leq 2$ and $0 \leq y \leq 6$, and there are no relations which x and y satisfy. Therefore, the double integral is just

$$\begin{aligned} \int_0^2 \int_0^6 2 - x\,dy dx &= 6 \int_0^2 2 - x\,dx \\ &= 6(2x - x^2/2)\Big|_{x=0}^{x=2} \\ &= 6(4 - 2) \\ &= 12. \end{aligned}$$

Therefore, the correct answer was (D). ◇

Problem 6.2. The absolute minimum value of

$$f(x, y) = 2 + x^2 y^2$$

in the region $x^2/2 + y^2 \leq 1$ is 2. Find the absolute maximum of f in this region.
(A) 4.5
(B) 4
(C) 3
(D) 2.5

(E) I don't know

Solution. By the extreme value theorem, we must check where the gradient of the function is 0 or at the boundary of the region $x^2/2 + y^2 \leq 1$. First, let us find the points where the derivative is 0. To do this, we take the gradient of f,

$$\operatorname{grad} f = \langle 2xy^2, 2x^2y \rangle.$$

It is clear that for $\operatorname{grad} f$ to equal $\langle 0, 0 \rangle$, either $x = 0$ or $y = 0$. This means that we must check the strips in Figure 1 which are highlighted in red. Along the x and y axis

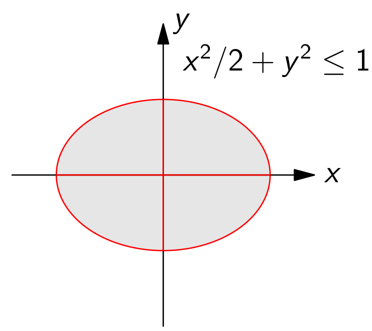

Figure 1: The region $x^2/2 + y^2 \leq 1$ with the critical region shaded in red.

(when either $x = 0$ or $y = 0$) the function f has the constant value $f = 2$, which we have been told is the minimum. Therefore, we must check the boundary, i.e., $x^2 + y^2 \leq 1$. Note that we can reparametrize the function f in terms of just x on the boundary by choosing either $y = \sqrt{1 - x^2/2}$ or $y = -\sqrt{1 - x^2/2}$. Let's choose the former (since we are ultimately interested in y^2, this choice won't matter). Then

$$f(x, \sqrt{1 - x^2/2}) = 2 + x^2(1 - x^2/2) = 2 + x^2 - x^4/2.$$

By the 1-variable version of the extreme value theorem, f achieves its maximum either on the boundary (the points $-\sqrt{2}$ or $\sqrt{2}$) or where the derivative vanishes, i.e.,

$$0 = f'(x) = 2x - 2x^3 = 2x(1 - x^2),$$

which is satisfied when $x = 0, x = \pm 1$. When $x = \pm 1$, we have

$$f(\pm 1, \sqrt{1 - 1/2}) = 2 + 1/2 = 3/2 = 2.5.$$

In every case, this will be the maximum.

The correct answer was (D). $\diamond$

Problem 6.3. Find the maximum of $2x + y$ on the circle $x^2 + y^2 = 10$.
(A) $3\sqrt{5}$
(B) $5\sqrt{2}$
(C) $\sqrt{30}$
(D) $2\sqrt{10}$
(E) I don't know
Hint: Set $f(x, y) = 2x + y$ and $g(x, y) = x^2 + y^2$ and use Lagrange multipliers.

Solution. Using the method of Lagrange multipliers, you can quickly come to the following equalities

$$2 = \lambda(2x),$$
$$1 = \lambda(2y).$$

From this we can deduce that $x = 2y$. Note that we do not need to find a value for λ, we merely need a relation between x and y. Substituting this back into our constraint, we get

$$f(2y, y) = 5y$$

and $5y^2 = 10$ which implies that $y = \pm\sqrt{2}$. Therefore, the maximum must happen at $y = \sqrt{2}$ since $f(2\sqrt{2}, \sqrt{2}) = 5\sqrt{2}$, whereas $f(-2\sqrt{2}, \sqrt{2})$.

The correct answer was (B). ◇