MA 261 Quiz 7
October 22, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 7.1. Evaluate the integral $\iint_D e^{-x^2-y^2}\,dA$, where D is the region bounded by the semicircle $x = \sqrt{25-y^2}$ and the y-axis

(A) $\pi(1-e^{-49})/2$
(B) $\pi(1-e^{-25})/2$
(C) $-\pi e^{-25}/2$
(D) $-\pi \sinh(25)/2$
(E) I don't know

Solution. To evaluate the integral above it is easiest to convert to polar coordinates first. In doing so, we arrive at the following equivalent integral

$$\iint_D re^{-r^2}\,dA = \int_0^{\pi}\int_0^{5} re^{-r^2}\,dr\,d\theta,$$

which, a moment's thought[1] we get

$$-\frac{\pi}{2}\left(e^{-25}-e^{0}\right) = \frac{\pi}{2}\left(1-e^{-25}\right).$$

Therefore, the correct answer was (B). ◇

Problem 7.2. Let R be the region in the first quadrant between the lines $y = 0$, $y = \sqrt{3}x$, and inside the circle $x^2+y^2 = 4$. Evaluate $\iint_R xy\,dA$.

(A) $1/2$
(B) $1/4$
(C) 2
(D) $3/2$
(E) I don't know

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[1]If you remember the Chain Rule from Calculus I, $d/dr(e^{-r^2}) = -2re^{-r^2}$ so the antiderivative of re^{-r^2} must be $-e^{-r^2}/2$

Solution. Like the previous problem, this problem is best tackled by transforming to polar coordinates, like so

$$\iint_R xy\, dA = \iint_R r^3 \sin\theta \cos\theta \, dr d\theta.$$

The next thing we need to do is find the bounds of our region R in terms of polar coordinates, which one quickly sees is $\theta = 0, \theta = \pi/3$ (since we go from $\tan\theta = 0/x = 0$ to $\tan\theta = \sqrt{3}x/x = \sqrt{3}$ and in $0 \le r^2 \le 4$). Thus, the integral becomes

$$\begin{aligned}
\int_0^{\pi/3} \int_0^2 r^3 \cos\theta \sin\theta \, dr d\theta &= \left(\int_0^2 r^3\, dr\right)\left(\int_0^{\pi/3} \sin\theta\cos\theta\, d\theta\right) \\
&= \left(\int_0^2 r^3\, dr\right)\left(\frac{1}{2}\int_0^{\pi/3} \sin(2\theta)\, d\theta\right) \\
&= \left(\frac{r}{4}\Big|_0^2\right)\left(-\frac{1}{4}\cos(2\theta)\Big|_0^{\pi/3}\right) \\
&= (1 - (-1/2)) = 3/2.
\end{aligned}$$

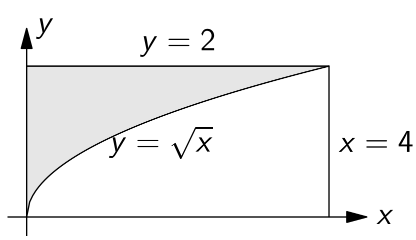
Therefore, the correct answer was (D). ◇

Problem 7.3. Compute the integral by changing the order of integration

$$\int_0^4 \int_{\sqrt{x}}^2 y \cos(y^4)\, dy dx$$

(A) $\sin(16)/4$
(B) $8\sin(16)/3$
(C) $2\sin(16)$
(D) $2\sin(16)/8$
(E) I don't know

Solution. To change the order of integration correctly, we need to sketch the region we are integrating over, and carefully select our bounds. You can verify for yourself that the region in question is the one shaded in gray in the figure below



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To change the order of integration, instead of traveling from $y = \sqrt{x}$ to $y = 2$, and $x = 0$ to $x = 4$ we travel from $x = 0$ to $x = y^2$, and $y = 0$ to $y = 2$. This gives us the necessary bounds to change the order of integration to

$$\begin{aligned}
\int_0^2 \int_0^{y^2} y \cos(y^4)\, dx dy &= \int_0^2 y^3 \cos(y^4)\, dy \\
&= \frac{1}{4} \sin(y^4)\Big|_{y=0}^{y=4} \\
&= \sin(16)/4.
\end{aligned}$$

Therefore, the correct answer was (A). $\diamond$