MA 261 Quiz 8
October 30, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number** and, just today, **five points** on top of that for writing both your name and section number.

Problem 8.1. Evaluate the integral

$$\iiint_E z\,dV$$

where E is the region in the first octant bounded by the sphere $x^2 + y^2 + z^2 = 1$.
Hint: Convert to spherical coordinates and apply the identity $\sin(2x) = 2\cos x \sin x$.

(A) $\pi/2$
(B) $\pi/12$
(C) $\pi/16$
(D) $\pi/4$
(E) I don't know

Solution. After converting to spherical coordinates, we may immediately start computing the integral, as follows:

$$\begin{aligned}
\iiint_E z\,dV &= \int_0^{\pi/2}\int_0^{\pi/2}\int_0^1 (\rho\cos\phi)\rho^2\sin\phi\,d\rho d\phi d\theta \\
&= \frac{\pi}{6}\int_0^{\pi/2}\cos\phi\sin\phi\,d\phi \\
&= \frac{\pi}{12}\int_0^{\pi/2}\sin(2\phi)\,d\phi \\
&= \frac{\pi}{12}\left[\frac{\cos(2\phi)}{2}\Big|_{\phi=0}^{\phi=\pi/2}\right] \\
&= \frac{\pi}{12}.
\end{aligned}$$

Therefore, the correct answer was (B). ◇

Problem 8.2. The equation of the surface $\phi = \pi/4$ converted to cylindrical coordinates becomes

Hint: Think about what the equation is telling you. The variables θ and ρ are allowed to change freely, but ϕ is fixed at $\pi/4$. What figure does that trace out?

(A) $z = 2r$
(B) $z = \sqrt{3}r$
(C) $z = r$
(D) $z = r/\sqrt{3}$
(E) I don't know

Solution. To convert to cylindrical coordinates, it is useful to know that $r = \rho \sin \phi$. Therefore,

$$z = \rho \cos(\pi/4) = \rho \sin(\pi/4) = r$$

so the surface we are after has equation $z = r$ in cylindrical coordinates. Therefore, the correct answer was (C). ◇