MA 261 Quiz 9
November 6, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 9.1. Find a function f (if possible) such that $\nabla f = \mathbf{F}$, where $\mathbf{F}(x, y, z) = \langle yz, xz, xy + 6z \rangle$

(A) $3z^2 + xyz$
(B) $3xyz^2$
(C) xyz
(D) DNE
(E) I don't know

Solution. The "if possible" was misleading as you did not know about curl yet. At any rate, checking against the given answer choices, we easily see that $\nabla(3z^2 + xyz) = \langle yz, xz, xy + 6z \rangle$. Therefore, the correct answer choice was (A). ◇

Problem 9.2. Find a function f such that $\nabla f(x, y, z) = \mathbf{F}(x, y, z) = \langle yz, xz, xy \rangle$ and use it to compute the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C = \{ x = t^3, y = 1 + t^2, z = (1 + t)^2, 0 \leq t \leq 1 \}$.

(A) 4
(B) 5
(C) 7
(D) 8
(E) I don't know

Solution. Although you did not know about curl, it is not necessary to check whether the given vector field $\mathbf{F}$ is conservative since you are told to find an f whose gradient is the given $\mathbf{F}$; in such a situation you should assume the given vector field is conservative.

It is not too difficult to see that $f(x, y, z) = xyz$ is a function whose gradient matches $\mathbf{F}$. Therefore, by the Fundamental Theorem of Line Integrals, after finding the endpoints of C (which are $(0, 1, 1)$ and $(1, 2, 4)$), we get

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(1, 2, 4) - f(0, 1, 1) = 8 - 0 = 8.$$

Therefore, the correct answer choice was (D). ◇

Problem 9.3. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = \langle\, y, -x, xy \,\rangle$ and C is parameterized by $\mathbf{r}(t) = \langle\, \sin t, \cos t, t \,\rangle$ with $0 \le t \le \pi$
(A) $\pi/2$
(B) $-\pi/2$
(C) π
(D) $-\pi$
(E) I don't know

Solution. In this case the field $\mathbf{F}$ is nonconservative (i.e., $\nabla \times \mathbf{F}(x, y, z) = \langle\, x, y, 0 \,\rangle$), but the line integral can be done explicitly, as follows. Let $x = \sin t$, $y = \cos t$, and $z = t$. Then

$$\begin{aligned}
\int_c \mathbf{F} \cdot d\mathbf{r} &= \int_0^\pi \langle\, \cos t, -\sin t, \sin t \cos t \,\rangle \cdot \langle\, \cos t, -\sin t, 1 \,\rangle \, dt \\
&= \int_0^\pi 1 + \sin t \cos t \, dt \\
&= \pi + \frac{1}{2} \int_0^\pi \sin(2t) \, dt \\
&= \pi + \frac{1}{4} \cos(2t) \Big|_{t=0}^{t=\pi} \\
&= \pi + \frac{1}{4}(1 - 1) \\
&= \pi.
\end{aligned}$$

Therefore, the correct answer was (C). ◇