MA 261 Quiz 11
April 9, 2019

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 11.1. Let $\mathbf{F}$ be a vector field and f a scalar field. Which of the following expressions are meaningful?

i. $\operatorname{curl} f$
ii. $\operatorname{div}(\operatorname{grad} f)$
iii. $(\operatorname{grad} f) \times (\operatorname{div} \mathbf{F})$
iv. $\operatorname{curl}(\operatorname{curl} \mathbf{F})$

(A) i only
(B) ii and iv only
(C) i, iii, and iv only
(D) iii only
(E) I don't know how to do this problem

Solution. We know from lecture that the curl operator takes a vector field $\mathbf{F}$ and returns a vector field $\operatorname{curl} \mathbf{F}$; the divergence operator takes a vector field $\mathbf{F}$ and returns a scalar field $\operatorname{div} \mathbf{F}$; and the gradient operator takes a scalar field f and returns a vector field $\operatorname{grad} f$. Following the chain of compositions in each of i, ii, ii, and iv, we see that only ii and iv make any since div takes a vector field and $\operatorname{grad} f$ returns a vector field, and, similarly, curl takes a vector field and $\operatorname{curl} \mathbf{F}$ returns a vector field.
Answer: (B) $\diamond$

Problem 11.2. Compute $\operatorname{div}(\operatorname{curl} \mathbf{F})$ for $\mathbf{F}(x, y, z) = yz^2\mathbf{i} + xy\mathbf{j} + yz\mathbf{k}$.
(A) 0
(B) 1
(C) 2
(D) 3
(E) I don't know how to do this problem

Solution. In the homework, you showed that $\operatorname{div}(\operatorname{curl} \mathbf{F}) = 0$ for any $\mathbf{F}$; if you knew this, you did not have to do any computation at all. Another useful identity is $\operatorname{curl}(\operatorname{grad} f) = \langle 0, 0, 0\rangle$; in fact, this comes from conservatism of

vector fields which you studied in the lecture, i.e., a vector field $\mathbf{F}$ is conservative if $\operatorname{curl}\mathbf{F} = \langle 0,0,0\rangle$ or, equivalently, if $\mathbf{F} = \operatorname{grad} f$ (these characterizations are really just coming from the fact that $\operatorname{curl}(\operatorname{grad} f) = \langle 0,0,0\rangle$).
Answer: (A) ◇

Problem 11.3. $\mathbf{F}(x,y,z) = yz\mathbf{i} + xz\mathbf{j} + xy\mathbf{k}$ is conservative, i.e., $\mathbf{F} = \operatorname{grad} f$ for some f. Find $\int_C \mathbf{F}\cdot d\mathbf{r}$ where C is the segment of the curve

$$\mathbf{r}(t) = t^3\mathbf{i} + (1+t^2)\mathbf{j} + (1+t)^2\mathbf{k}$$

from $0 \le t \le 1$.
Hint: By the Fundamental Theorem of Line Integrals, $\int_C \mathbf{F}\cdot d\mathbf{r} = f(b) - f(a)$ where a is the starting point of C and b the end point.
(A) 4
(B) 5
(C) 7
(D) 8
(E) I don't know how to do this problem

Solution. By the Fundamental Theorem of Line Integrals, all we need to do is determine a suitable scalar f field; one such that $\mathbf{F} = \operatorname{grad} f$. It is quite easy to see that the scalar field

$$f(x,y,z) = xyz$$

is one such field. To finish the problem, we need to determine the starting and ending points for the curve segment C. These are easy enough to determine from the parametrization:

$$r(0) = \langle 0,1,1\rangle,$$
$$r(1) = \langle 1,2,4\rangle.$$

Lastly, by the FTLI,

$$\int_C \mathbf{F}\cdot d\mathbf{r} = f(1,2,4) - f(0,1,1) = 1\cdot 2\cdot 4 - 0\cdot 1\cdot 1 = 8.$$

Answer: (D) ◇