MA 261 Quiz 2
January 22, 2019

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 2.1. At what points does the curve $\mathbf{r}(t) = \langle t, 0, 2t - t^2 \rangle$ intersect the paraboloid $z = x^2 + y^2$?
(A) $(0, 0, 0), (1, 0, 1)$
(B) $(2, 0, 4), (2, 0, 0)$
(C) $(-1, 0, -3), (1, 1, 2)$
(D) $\left(1, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right), (0, 1, 1)$
(E) I don't know how to do this

Solution. For the curve $\mathbf{r}$ to intersect the paraboloid the coordinates of the curve $x(t) = t$, $y(t) = 0$, and $z(t) = 2t - t^2$ must satisfy the defining equation of the paraboloid, i.e.,

$$\begin{aligned} z(t) &= x(t)^2 + y(t)^2 \\ (2t - t^2) &= t^2 + 0^2 \\ (2t - t^2) - t^2 &= 0 \\ 2t - 2t^2 &= 0 \\ t(1 - t) &= 0. \end{aligned}$$

Therefore, at $t = 0$ and $t = 1$, the curve $\mathbf{r}$ intersects the paraboloid, and the points corresponding to $t = 0$ and $t = 1$ are

$$\begin{aligned} \mathbf{r}(0) &= \langle 0, 0, 0 \rangle \\ \mathbf{r}(1) &= \langle 1, 0, 1 \rangle. \end{aligned}$$

Therefore, the correct answer choice was (A).

This problem could be gamed by looking at the answer choices and figuring out which of the choices meet the conditions of being both a point traversed by $\mathbf{r}$ and a point in the paraboloid.

Answer: (A). ◇

Problem 2.2. What does the equation

$$x^2 - 2y^2 + z^2 = -1$$

represent as a surface in $\mathbf{R}^3$?
(A) elliptic paraboloid
(B) hyperboloid of one sheet
(C) hyperboloid of two sheets
(D) hyperbolic paraboloid
(E) I don't know how to do this

Solution. Problems like these are most easily dealt with by memorizing the name and form of quadratic surfaces. The equation $x^2 - 2y^2 + z^2 = -1$ most closely resembles that of a hyperboloid of two sheets. And indeed this is the case.
Answer: (C). ◇

Problem 2.3. Two particles travel along the curves

$$\mathbf{r}_1(t) = \langle t, t^2 + 1, -t \rangle, \text{ and } \mathbf{r}_2(t) = \langle 1 + 2t, 3 + t, 4 + 3t \rangle.$$

What is their first point of collision?
(A) $(-1, 2, 1)$
(B) $(1, 2, -1)$
(C) $(1, 3, 4)$
(D) the particles do not collide
(E) I don't know how to do this

Solution. There was some ambiguity to this problem, as pointed out by some of you. I did not say that the particle started moving at $t = 0$; if this were the case, as far as we know, the curves $\mathbf{r}_1$ and $\mathbf{r}_2$ do not accurately describe the trajectory taken by the particles for $t < 0$. In particular, it would not make sense to say that the particles collided at $t = -1$ at the point $(-1, 2, 1)$ (which was the intended solution). Instead, as t goes forward, the two particles *never collide*.
Answer: Both (A) and (D) were counted as correct. ◇