MA 261 Quiz 3
January 29, 2019

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 3.1. Consider the curve $\mathbf{r}(t) = \langle t, 3\sin t, 3\cos t\rangle$. Find $\mathbf{r}'(t)$

(A) $\mathbf{r}'(t) = \langle 1, 3\cos t, -3\sin t\rangle$
(B) $\mathbf{r}'(t) = \langle t, 3\sin t, 3\sin t\rangle$
(C) $\mathbf{r}'(t) = \langle 1, 3, -3\rangle$
(D) $\mathbf{r}'(t) = \langle 1, 3, 3\rangle$
(E) I don't know how to do this

Solution. This first problem is easy if you remember how to take derivatives of trigonometric functions,

$$\mathbf{r}'(t) = \langle 1, 3\cos t, -3\sin t\rangle$$

Answer: (A). ◇

Problem 3.2. Find the arclength of $\mathbf{r}(t) = \langle t, 3\sin t, 3\cos t\rangle$ for $0 \le t \le 1$?

(A) $\sqrt{10}$
(B) 3
(C) $\sqrt{3}/2$
(D) 3π
(E) I don't know how to do this

Solution. By the arclength formula,

$$\begin{aligned} l(0,1) &= \int_0^1 |\mathbf{r}'(t)|\,dt \\ &= \int_0^1 \sqrt{1 + 9\sin^2 t + 9\cos^2 t}\,dt \\ &= \int_0^1 \sqrt{1+9}\,dt \\ &= \int_0^1 \sqrt{10}\,dt. \end{aligned}$$

Answer: (A). $\diamond$

Problem 3.3. Find the curvature of $\mathbf{r}(t) = \langle t, 3\sin t, 3\cos t\rangle$?
(A) $3/\sqrt{10}$
(B) $1/3$
(C) 1
(D) $3/10$
(E) I don't know how to do this

Solution. Recall that the curvature κ of a curve $\mathbf{r}$ is the same as the magnitude of the second derivative of the arclength parametrized form of $\mathbf{r}$, so all we need to do is find the arclength parametrization of $\mathbf{r}$ in the statement of the problem and find its second derivative.

From the last problem, it is easy to arclength parametrize $\mathbf{r}(t)$ using the equality $s = \sqrt{10}t$, i.e.,

$$\begin{aligned}
\mathbf{r}(s) &= \left\langle \frac{s}{\sqrt{10}}, 3\sin\left(\frac{s}{\sqrt{10}}\right), 3\cos\left(\frac{s}{\sqrt{10}}\right)\right\rangle \\
\mathbf{r}'(s) &= \left\langle \frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}\cos\left(\frac{s}{\sqrt{10}}\right), -\frac{3}{\sqrt{10}}\sin\left(\frac{s}{\sqrt{10}}\right)\right\rangle \\
\mathbf{r}''(s) &= \left\langle 0, -\frac{3}{10}\sin\left(\frac{s}{\sqrt{10}}\right), -\frac{3}{10}\cos\left(\frac{s}{\sqrt{10}}\right)\right\rangle.
\end{aligned}$$

Therefore,

$$\kappa = |\mathbf{r}''(s)| = \frac{3}{10}.$$

Answer: (D). $\diamond$