MA 261 Quiz 5
February 12, 2019

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 5.1. Find an equation for the plane tangent to $xy^2z^3 = 12$ at $(3, 2, 1)$.
(A) $x + 2y + 3z = 10$
(B) $x + y + z = 6$
(C) $3x + 2y + z = 14$
(D) $x + 3y + 9z = 18$
(E) I don't know how to do this

Solution. As we mentioned in class, to find the plane tangent to $xy^2z^3 = 12$ at the point $(3, 2, 1)$, we need to find the gradient of the function $F(x, y, z) = xy^2z^3 - 12$ at the point $(3, 2, 1)$ and then put it into the tangent plane formula

$$\text{grad } f \cdot \langle x - x_0, y - y_0, z - z_0 \rangle = 0.$$

First, it is easy to compute that $\text{grad } f(x, y, z) = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$ so $\text{grad } f(3, 2, 1) = \langle 4, 12, 36 \rangle$. Therefore, the equation for the tangent plane is

$$\langle 4, 12, 36 \rangle \cdot \langle x - 3, y - 2, z - 1 \rangle = 0,$$

or, if we simplify this,

$$x + 3y + 9z = 18.$$

Answer: (D). ◇

Problem 5.2. Use linear approximation on $f(x, y) = \sqrt{x^2 + 3y}$ at $(4, 3)$ to approximate the value of $\sqrt{(4.02)^2 + 3(2.97)}$.
(A) 5.004
(B) 5.05
(C) 4.093
(D) 5.007
(E) I don't know how to do this

Solution. Recall from class that to find the linear approximation of a function f we must find its tangent line. That is, using f as given above, first we compute its gradient

$$\operatorname{grad} f(x,y) = \frac{\left\langle x, 3/2 \right\rangle}{\sqrt{x^2+3y}}.$$

Therefore, the tangent line at the point $(4,3)$ is given by

$$L(x,y) = \frac{4}{5}(x-4) + \frac{3}{10}(y-3) + 5,$$

and thus,

$$\begin{aligned}\sqrt{(4.02)^2 + 3(2.97)} &\approx L(4.02, 2.97)\\ &= 5 + (0.8)(0.02) - (0.3)(0.03)\\ &= 5.007.\end{aligned}$$

Answer: (D) ◇

Problem 5.3. Find the directional derivative of $f(x,y) = \sin(4x+2y)$ at the point $(-4,8)$ in the direction $\mathbf{v} = \sqrt{3}\mathbf{i} - \mathbf{j}$.

(A) 0
(B) $4\sqrt{3} - 2$
(C) $2\sqrt{3} - 1$
(D) $2 - 4\sqrt{3}$
(E) I don't know how to do this

Solution. By the directional derivative formula, first we need to find the unit direction, $\mathbf{u}$, of $\mathbf{v}$, which is

$$\mathbf{u} = \frac{\mathbf{v}}{|\mathbf{v}|} = \frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}.$$

We also need the gradient at $(-4,8)$, which is

$$\operatorname{grad} f(x,y) = \langle 4\cos(4x+2y), 2\cos(4x+2y)\rangle,$$

so

$$\operatorname{grad} f(-4,8) = 4\mathbf{i} + 2\mathbf{j}.$$

Therefore,

$$D_{\mathbf{u}} f(-4,8) = \left(\frac{\sqrt{3}}{2}\mathbf{i} - \frac{1}{2}\mathbf{j}\right) \cdot (4\mathbf{i} + 2\mathbf{j}) = 2\sqrt{3} - 1.$$

Answer: (C). ◇