MA 261 Quiz 7
March 5, 2018

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

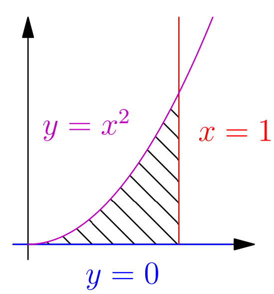
Problem 7.1. Let D be the region bounded by $y = x^2$, $y = 0$, and $x = 1$. If the density is $\rho(x, y) = x$, find $\bar{y}$

(A) 1/3
(B) 1/4
(C) 2/5
(D) 3/4
(E) I don't know how to do this

Solution. Recall that $\bar{y} = M_x/M$, where

$$M = \iint_D \rho(x, y)\, dA, \text{ and } M_x = \iint_D y\rho(x, y)\, dA.$$

No, if we sketch the region, as we do below



it is not too difficult to see that the bounds for the double integrals are $0 \le x \le 1$ and $0 \le y \le x^2$. Thus,

$$\begin{aligned} M &= \int_0^1 \int_0^{x^2} x\, dydx, \\ &= \int_0^1 x^3\, dx \\ &= \frac{1}{4} \end{aligned} \qquad \begin{aligned} M_x &= \int_0^1 \int_0^{x^2} yx\, dydx \\ &= \frac{1}{2} \int_0^1 x^5\, dx \\ &= \frac{1}{12} \end{aligned}$$

so

$$\bar{y} = M_x/M = \frac{1/12}{1/4} = \frac{4}{12} = \frac{1}{3}.$$

Answer: (A). ◇

Problem 7.2. Compute $\iint_D e^{x^2+y^2}\, dA,\ D = \{x^2 + y^2 \leq 1\}$ by changing to polar coordinates.
To clarify, D is the unit disk centered at the origin.
(A) 1
(B) $\pi(e-1)$
(C) e^{π}
(D) πe
(E) I don't know how to do this

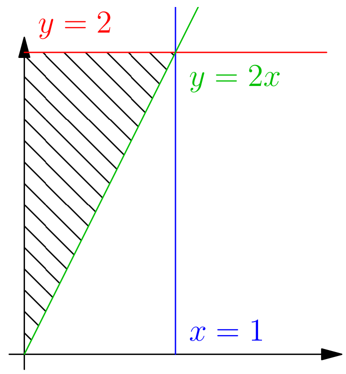
Solution. By changing to polar coordinates, the computation follows easily:

$$\begin{aligned}\iint_D e^{x^2+y^2}\, dA &= \int_0^{2\pi}\int_0^1 re^{r^2}\, drd\theta \\ &= 2\pi\int_0^1 re^{r^2}\, dr \\ &= 2\pi\Big[e^{r^2}/2\Big]_0^1 \\ &= \pi(e-1).\end{aligned}$$

Answer: (B). ◇

Problem 7.3. Compute $\int_0^1\int_{2x}^2 e^{y^2}\, dydx$ by reversing the order of integration.
(A) $e^4/4$
(B) e^4
(C) e^4-1
(D) $e^4/4-1/4$
(E) I don't know how to do this

Solution. First we sketch the region of integration, as below



Having done this, it is quite easy to see that

$$\int_0^1 \int_{2x}^2 e^{y^2}\, dydx = \int_0^2 \int_0^{y/2} e^{y^2}\, dxdy$$
$$= \frac{1}{2} \int_0^2 ye^{y^2}\, dy$$

(by u-substitution, with $u = y^2$)

$$= \frac{1}{4} \Big[e^{y^2} \Big]_0^2$$
$$= e^4/4 - 1/4.$$

Answer: (D) $\diamond$