MA 261 Quiz 8
March 19, 2019

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number** and, just today, **five points** on top of that for writing both your name and section number.

Problem 8.1. Evaluate the triple integral $\iiint_E 8xyz\, dV$, where

$$E = \{2 \le x \le 3, 1 \le y \le 2, 0 \le z \le 1\}.$$

(A) 2
(B) 4
(C) 10
(D) 15
(E) I don't know how to do this

Solution. The integral in this problem can be directly computed as is:

$$\begin{aligned}\int_2^3 \int_1^2 \int_0^1 8xyz\, dzdydx &= 8\left[\int_2^3 x\, dx\right]\left[\int_1^2 y\, dy\right]\left[\int_0^1 z\, dz\right] \\ &= 8\Big[x^2/2\Big]_{x=2}^{x=3}\Big[y^2/2\Big]_{y=1}^{y=2}\Big[z^2/2\Big]_{z=0}^{z=1} \\ &= (3^2 - 2^2)(2^2 - 1^2)(1^2 - 0^2) \\ &= (5)(3)(1) \\ &= 15.\end{aligned}$$

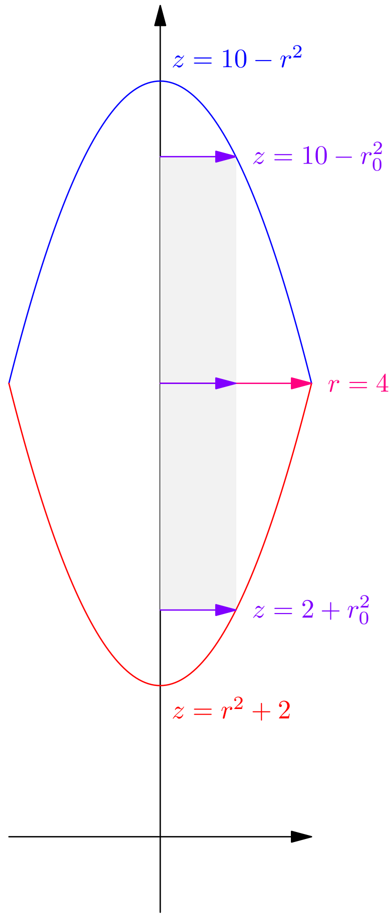
Answer: (B). $\diamond$

Problem 8.2. Let E be the region bounded by two surfaces whose equations in cylindrical coordinates are $z = 10 - r^2$ and $z = 2 + r^2$. Find the volume of E.

(A) 8π
(B) 12π
(C) 16π
(D) 18π

(E) I don't know how to do this

Solution. As the figure illustrates



the simplest way to find the volume is by integrating $\iiint_E dV$ in the order $dzdrd\theta$ with

$$r^2 + 2 \leq z \leq 10 - r^2, \quad 0 \leq r \leq 4, \quad 0 \leq \theta \leq 2\pi.$$

Thus

$$\begin{aligned}\iiint_E dV &= \int_0^{2\pi}\int_0^2\int_{2+r^2}^{10-r^2} r\,dzdrd\theta \\ &= \left[\int_0^{2\pi} d\theta\right]\left[\int_0^2\int_{2+r^2}^{10-r^2} r\,dzdr\right] \\ &= 2\pi\int_0^2 (10-r^2-2-r^2)r\,dr \\ &= 2\pi\int_0^2 8r-2r^3\,dr \\ &= 2\pi\Big[4r^2-r^4/2\Big]_0^2 \\ &= 16\pi.\end{aligned}$$

Answer: (B) $\diamond$

Problem 8.3. Rewrite the iterated integral

$$\int_0^1\int_0^{1-x}\int_0^{2-2y} f(x,y,z)\,dzdydx$$

by changing the order of integration to $dxdydz$.

(A) $\int_0^2\int_0^{1-z/2}\int_0^{y-1} f(x,y,z)\,dxdydz$
(B) $\int_0^2\int_0^{1-x}\int_0^{1-y} f(x,y,z)\,dxdydz$
(C) $\int_0^2\int_0^{2-2x}\int_0^{1-y} f(x,y,z)\,dxdydz$
(D) $\int_0^2\int_0^{1-z/2}\int_0^{1-y} f(x,y,z)\,dxdydz$
(E) I don't know how to do this

Solution. An excellent starting point for this problem is to write out the bounds of the integrals. These are

$$0\le x\le 1,\quad 0\le y\le 1-x,\quad 0\le z\le 2-2y.$$

We want to change the order of integration from $dzdydx$ to $dxdydz$. In order to do this, we need to find what is the largest possible value z can take, and subject to a chosen z, what is the largest possible value y can take, and so on. To do this, note that

$$0\le z\le 2-2y\le 2-2(1-x)\le 2-2(1-1)=2.$$

So maximizing the inequality above (the case where the inequality is in fact a chain of equalities), we get

$$0\le z\le 2.$$

Similarly, for y,

$$0 \leq z \leq 2 - 2y$$

gives us

$$0 \leq y \leq 2 - \frac{z}{2}.$$

Lastly, since x only depends on y as in the inequality

$$0 \leq y \leq 1 - x,$$

we get

$$0 \leq x \leq 1 - y.$$

Therefore,

$$\int_0^1 \int_0^{1-x} \int_0^{2-2y} f(x, y, z)\, dzdydx = \int_0^2 \int_0^{1-z/2} \int_0^{1-y} f(x, y, z)\, dxdydz.$$

Answer: (D). $\diamond$