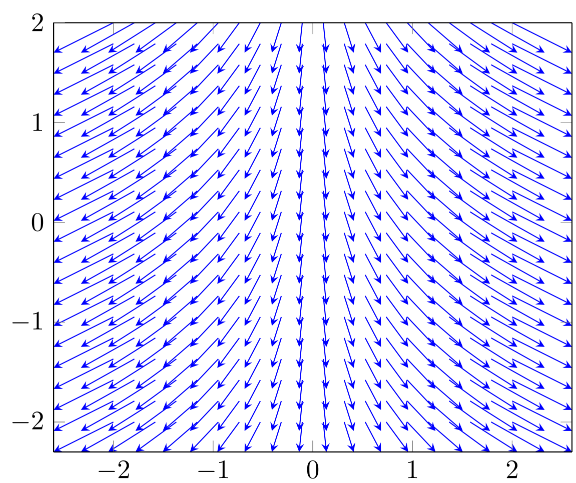
MA 261 Quiz 9
March 26, 2019

If you do not know how to do any one of these problems, circle **"(E) I don't know"** as your answer choice. You will receive **two points** for doing that. **Each problem** is worth **five points**. You get **two points** for writing your **full name** and **three points** for writing your **section number**.

Problem 9.1. The graph below most closely resembles which of the following vector fields?



(A) $2x\mathbf{i} - 2\mathbf{j}$
(B) $\mathbf{i} + (x - y)\mathbf{j}$
(C) $-(y/x^2)\mathbf{i} + (1/x)\mathbf{j}$
(D) $2x\mathbf{i} + 2y\mathbf{j}$
(E) I don't know how to do this problem

Solution. The only reasonable solution to this is $2x\mathbf{i} - 2\mathbf{j}$. In the image, at each point, the vector field is pointing downwards. This means that the $\mathbf{j}$ component of the vector field is negative. For the other options, the $\mathbf{j}$ component is not only changing, but it changes sign, i.e. it goes from positive to negative and so we should expect to see some vectors pointing upward in the grid, but we do not.
Answer: (A) $\diamond$

Problem 9.2. Evaluate the line integral $\int_C 4y\,dx + 5z\,dy + 3x\,dz$, where C is the curve $\mathbf{r}(t) = t\mathbf{i} + t^3\mathbf{j} + t^2\mathbf{k}$ for $0 \le t \le 1$.
(A) 3
(B) 4
(C) 5
(D) 6
(E) I don't know how to do this

Solution. By the Chain Rule,

$$dx = dt, \quad dx = 3t^2\,dt, \quad dz = 2t\,dt.$$

Now, applying the formula for the line integral

$$\begin{aligned}
\int_C 4y\,dx + 5z\,dy + 3x\,dz &= \int_0^1 4(t^3) + 5t^2(3t^2) + 3t(2t)\,dt \\
&= \int_0^1 4t^3 + 15t^4 + 6t^2\,dt \\
&= \left[t^4 + 3t^5 + 2t^3\right]_0^1 \\
&= 1 + 3 + 2 = 6.
\end{aligned}$$

Answer: (D). ◇

Problem 9.3. Evaluate the integral $\iiint_E \sqrt{x^2+y^2+z^2}\,dV$, where E is the region above the cone $\sqrt{3}z = \sqrt{x^2+y^2}$ and below the sphere $x^2+y^2+z^2 = 10$.
(A) 5π
(B) 25π
(C) $50\pi(1 - \sqrt{3}/2)$
(D) $50\pi(1 - \sqrt{2}/2)$
(E) I don't know how to do this

Solution. The problem is made easy by first converting from rectangular to spherical coordinates. Upon making the conversion, we see that

$$0 \le \theta \le 2\pi, \quad 0 \le \rho \le \sqrt{10}, \quad 0 \le \phi \le \pi/3.$$

Therefore,

$$\begin{aligned}
\iiint_E \sqrt{x^2+y^2+z^2}\,dV &= \int_0^{2\pi}\int_0^{\pi/3}\int_0^{\sqrt{10}} \rho^3 \sin\phi\, d\rho d\phi d\theta \\
&= \left[\int_0^{2\pi} d\theta\right]\left[\int_0^{\pi/3} \sin\phi\, d\phi\right]\left[\int_0^{\sqrt{10}} \rho^3\, d\rho\right] \\
&= 2\pi(-1/2+1)(10^2/4) \\
&= 25\pi
\end{aligned}$$

Answer: (B). $\diamond$