STAT 472
Chapter 2 and 3 Formula Sheet

$$S_0(x)=1-F_0(x)={}_xp_0=1-{}_xq_0=e^{-\int_0^x \mu_s ds}=\frac{l_x}{l_0}$$

$$S_x(t)=1-F_x(t)=\frac{S_0(x+t)}{S_0(x)}={}_tp_x=1-{}_tq_x=\frac{{}_{x+t}p_0}{{}_xp_0}=e^{-\int_x^{x+t}\mu_r dr}=e^{-\int_0^t \mu_{x+s}ds}=\frac{l_{x+t}}{l_x}$$

$$p_x=1-q_x=\frac{l_{x+1}}{l_x}$$

$$S_x(t+u)=S_x(t)\cdot S_{x+t}(u)={}_{t+u}p_x={}_tp_x\cdot {}_up_{x+t}=\frac{l_{x+t+u}}{l_x}$$

$${}_tq_x=F_x(t)=1-S_x(t)=1-{}_tp_x=\frac{l_x-l_{x+t}}{l_x}$$

$$q_x=1-p_x=\frac{l_x-l_{x+1}}{l_x}=\frac{d_x}{l_x}$$

$${}_tp_x+{}_tq_x=1$$

$$l_{x+1}=l_x\cdot p_x$$

$$d_x=l_x-l_{x+1}=l_x\cdot q_x$$

$${}_{u|t}q_x={}_up_x-{}_{u+t}p_x={}_up_x\cdot {}_tq_{x+u}=\frac{l_{x+u}-l_{x+u+t}}{l_x}$$

μ_x=force of mortality= $\dfrac{-\frac{d}{dx}S_0(x)}{S_0(x)}=-\dfrac{d}{dx}\ln[S_0(x)]=\dfrac{f_0(x)}{S_0(x)}=\dfrac{-\frac{d}{dx}\,{}_xp_0}{{}_xp_0}$

$$\mu_{x+t}=\frac{-\frac{d}{dt}S_x(t)}{S_x(t)}=-\frac{d}{dt}\ln[S_x(t)]=\frac{f_x(t)}{S_x(t)}=\frac{-\frac{d}{dt}\,{}_tp_x}{{}_tp_x}$$

$$f_x(t)=\frac{d}{dt}F_x(t)=-\frac{d}{dt}S_x(t)=S_x(t)\cdot\mu_{x+t}={}_tp_x\cdot\mu_{x+t}$$

$$E[T_x]=\overset{o}{e}_x=complete\ expectation\ of\ life=\int_o^\infty t\cdot {}_tp_x\cdot\mu_{x+t}\cdot dt=\int_0^\infty {}_tp_x\cdot dt$$

$$E[T_x^2]=\int_o^\infty t^2\cdot {}_tp_x\cdot\mu_{x+t}\cdot dt=2\int_0^\infty t\cdot {}_tp_x\cdot dt$$

$$Var[T_x]=E[T_x^2]-(E[T_x])^2$$

$$E[\min(T_x,n)]=\overset{o}{e}_{x:\overline{n}|}=n-year\ partial\ or\ term\ complete\ expectation\ of\ life=\int_o^n t\cdot {}_tp_x\cdot\mu_{x+t}\cdot dt=\int_0^n {}_tp_x\cdot dt$$

$$E[\{\min(T_x,n)\}^2]=\int_o^n t^2\cdot {}_tp_x\cdot\mu_{x+t}\cdot dt=2\int_0^n t\cdot {}_tp_x\cdot dt$$

$$Prob[K_x=k]={}_{k|}q_x={}_kp_x-{}_{k+1}p_x={}_kp_x\cdot q_{x+k}$$

$$E[K_x]=e_x=curtate\ expectation\ of\ life=\sum_{k=0}^\infty k({}_kp_x-{}_{k+1}p_x)=\sum_{k=1}^\infty {}_kp_x$$

$$E[K_x^2]=\sum_{k=0}^\infty k^2({}_kp_x-{}_{k+1}p_x)=\{2\sum_{k=1}^\infty k\cdot {}_kp_x\}-e_x$$

$$Var[K_x]=E[K_x^2]-(E[K_x])^2$$

$$E[\min(K_x,n)]=e_{x:\overline{n}|}=n-year\ term\ curtate\ expectation\ of\ life=\sum_{k=1}^n {}_kp_x$$

$$E[\{\min(K_x,n)\}^2]=\{2\sum_{k=1}^n k\cdot {}_kp_x\}-e_{x:\overline{n}|}$$

$\overset{o}{e}_x\approx e_x+\frac{1}{2}$ <= This is exact under UDD and approximate otherwise.

$$e_x={}_1p_x(1+e_{x+1})$$

$$e_x=e_{x:\overline{n-1}|}+{}_np_x(1+e_{x+n})=e_{x:\overline{n}|}+{}_np_x\cdot e_{x+n}$$

$$\overset{o}{e}_x=\overset{o}{e}_{x:\overline{n}|}+{}_np_x\cdot\overset{o}{e}_{x+n}$$

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	Gompertz	Makeham
μ_x	Bc^x	$A+Bc^x$
${}_t p_x$	$\exp\left\{\frac{-B}{\ln(c)}(c^x)(c^t-1)\right\}$	$\exp\left\{-A\cdot t-\frac{B}{\ln(c)}(c^x)(c^t-1)\right\}$

UDD for $0\le s\le 1$	**CFM for** $0\le s\le 1$
$l_{x+s}=(1-s)l_x+(s)l_{x+1}$	$l_{x+s}=l_x\cdot(p_x)^s=(l_x)^{1-s}\cdot(l_{x+1})^s$
${}_s q_x=s\cdot q_x$	${}_s p_x=(p_x)^s$
$l_{x+s}=l_x-s\cdot d_x$	${}_s p_{x+t}=(p_x)^s\ \textit{if}\ s>0\ \textit{and}\ t+s\le 1$
UDD for $0\le s<1$	**CFM for** $0\le s<1$
$\mu_{x+s}=\frac{q_x}{1-s\cdot q_x}$	$\mu_{x+s}=-\ln[p_x]$