Topic: Accumulation Function, Simple & Compound Interest, Effective Interest Rate

Britney invests 10,000 in an account earning interest based on an accumulation function of $\alpha + \beta t^2$. After 3 years, Britney has 11,800.

Determine the effective interest rate that Britney earned during the third year. This would be $i_{[2,3]}$ in symbols.

Solution:

$$a(t) = \alpha + \beta t^2$$

$$a(0) = 1 = \alpha + \beta(0^2) \Rightarrow \alpha = 1$$

$$(10,000)a(3) = 10,000(\alpha + \beta(3^2)) = 11,800$$

$$11,800 = 10,000 + 90,000\beta$$

$$1,800 = 90,000\beta \Rightarrow \beta = 0.02$$

$$\begin{aligned} i_{[2,3]} &= \frac{a(3) - a(2)}{a(2)} \\ &= \frac{(1 + 0.02(3^2)) - (1 + 0.02(2^2))}{(1 + 0.02(2^2))} \\ &= 0.092592593 \end{aligned}$$

Alan invests 10,000 in an account earning simple interest. At the end of 20 years, Alan has 30,000.

Kelly invests 10,000 in an account earning compound interest at an annual effective interest rate of i .

During the 11th year, Alan and Kelly earn the same annual effective interest rate.

Determine the amount that Kelly has at the end of the 20th year.

Solution:

Alan:

$$10{,}000(1+20s)=30{,}000 \quad ==> \quad 10{,}000+200{,}000s=30{,}000$$

$$==> \quad 1+20s=3 \quad ==> \quad 20s=2 \quad ==> \quad s=0.1$$

$$i_{11}^{Alan}=i_{11}^{Kelly}$$

$$i_{11}^{Alan}=\frac{s}{1+(n-1)s}=\frac{0.1}{1+(11-1)(0.1)}=0.05$$

$$i_{11}^{Kelly}=i=0.05$$

$$Amount=(10{,}000)(1+0.05)^{20}=26{,}532.98$$

Kaitlyn invests 10,000 in an account earning simple interest. At the end of 10 years, Kaitlyn has 20,000.

Jalen invests 10,000 in an account earning compound interest.

The effective interest rate during the tenth year for Kaitlyn is equal to the effective interest rate during the tenth year for Jalen.

Determine the amount that Jalen has in his account at the end of 10 years.

Solution:

Kaitlyn

$$10,000(1+10s) = 20,000 \implies 1+10s = 2 \implies 10s = 1 \implies s = 0.1$$

$$i_{10}^{Kaitlyn} = i_{10}^{Jalen}$$

$$i_{10}^{Kaitlyn} = \frac{s}{1+(n-1)s} = \frac{0.1}{1+9(0.1)} = 0.052631579$$

$$i_{10}^{Jalen} = i = 0.052631579$$

$$Amount = (10,000)(1+0.052631579)^{10} = 16,701.83$$

Jordyn invests 1000 in Bank Chen. Bank Chen pays simple interest rate of s. In the 10th year, Jordyn earns an annual effective interest rate of 5%.

Calculate the amount of money Jordyn will have at the end of the 10th year.

Solution:

By Definition

The effective interest rate in the nth year $= i_n = \dfrac{a(n)-a(n-1)}{a(n-1)}$

Which for simple interest is

$$\frac{a(n)-a(n-1)}{a(n-1)} = \frac{\left(1+(n)s\right)-\left(1+(n-1)s\right)}{1+(n-1)s} = \frac{1+ns-1-ns+s}{1+(n-1)s} = \frac{s}{1+(n-1)s}$$

$$i_{10} = 0.05 = \frac{s}{1+(10-1)(s)} = \frac{s}{1+9s} \text{ ==> } 0.05(1+9s) = s$$

$$\text{==> } 0.05+0.45s = s \text{ ==> } 0.05 = 0.55s \text{ ==> } s = \frac{1}{11}$$

$$A(10) = (1000)\left[1+\left(\frac{1}{11}\right)(10)\right] = 1909.09$$

Niu invests 1000 in an account that earns compound interest. At the end of 12 years, Niu has 3138.43.

Yang invests 1000 in an account that earns simple interest for 12 years.

During the sixth year, Niu and Yang earn the same annual effective interest rate.

Determine the amount the Yang will have at the end of 12 years.

Solution:

$$\text{Niu} ==> 1000(1+\text{i})^{12} = 3138.43 ==> (1+i)^{12} = 3.13843 ==> i = 0.10$$

$$i_6^{Compound} = i_6^{Simple}$$

$$0.10 = \frac{s}{1+(6-1)s} ==> 0.10 + 0.5s = s ==> 0.10 = 0.5s ==> s = 0.2$$

$$\text{Yang} ==> 1000(1+(12)(0.2)) = 3400$$

Brandon borrows 4000 at a simple interest rate. At the end of 8 years, Brandon repays the loan with a payment of 5800.

Calculate the effective interest rate for the last two years of the loan which is $i_{[6,8]}$.

Solution:

$$a(t) = 1 + st$$

$$4000(1+8s) = 5800 ==> 4000 + 32{,}000s = 5800$$

$$==> s = \frac{5800-4000}{32{,}000} = 0.05625$$

$$i_{[6,8]} = \frac{a(8)-a(6)}{a(6)} = \frac{1+(0.05625)(8) - [1+(0.05625)(6)]}{1+(0.05625)(6)} = \frac{1.45-1.3375}{1.3375} = 0.08411$$