Topic: Annuities Immediate and Due

Brenna wants to have 500,000 on her 40th birthday. Today is her 20th birthday. She will make a deposit of P into an account at the beginning of each month for the next 20 years. The account will earn an annual effective interest rate of 8%.

Determine P so that Brenna will have exactly 500,000 on her 40th birthday.

Solution:

Since payments are monthly, we need the monthly effective interest rate of $\frac{i^{(12)}}{12}$.

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = 1+i = 1.08 \quad ==> \quad \frac{i^{(12)}}{12} = (1.08)^{\frac{1}{12}} - 1 = 0.006434$$

$$P\ddot{s}_{\overline{240|}} = 500{,}000 \quad ==> \quad P\left(\frac{(1.006434)^{240}-1}{0.006434}\right)(1.006434) = 500{,}000$$

$$==> \quad 572.66P = 500{,}000 \quad ==> \quad P = 873.12$$

Kiran wants to have 1,000,000 when she turns 65. Today is Kiran's 25th birthday. In order to accomplish her goal, she will deposit D into an account earning an annual effective interest rate of 8% at the beginning of each year for the next 40 years.

Determine D.

Solution:

$$D\ddot{s}_{\overline{40|}} = 1,000,000$$

$$D = \frac{1,000,000}{\left(\frac{(1.08)^{40}-1}{0.08}\right)(1.08)} = 3574.22$$

Or

Set BGN

$\boxed{N} \leftarrow 40$

$\boxed{I/Y} \leftarrow 8$

$\boxed{FV} \leftarrow 1,000,000$

$\boxed{CPT}\boxed{PMT} \Rightarrow 3574.22$

Emily is saving for her retirement. She invests 100 at the beginning of each month for 40 years into an account earning an annual effective interest rate of 9%.

Calculate the amount that Emily will have at the end of 40 years.

Solution:

$$Amount = (100)\ddot{s}_{\overline{480}|}$$

Since payments are monthly, we need the monthly effective interest rate of $\frac{i^{(12)}}{12}$.

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = 1+i = 1.09 ==> \frac{i^{(12)}}{12} = (1.09)^{1/12} - 1 = 0.007207323$$

$$(100)\ddot{s}_{\overline{480}|} = (100)\left(\frac{(1.007207323)^{480} - 1}{0.007207323}\right)(1.007207323) = 424{,}964.87$$

Kayla borrows 10,000 from Alex. Alex tells Kayla that she can repay him using one of the following two options:

a. Pay a single payment 26,017.40 at the end of 10 years; or

b. Make monthly payments of P for the next 10 years.

Both payment options are equivalent which means that Kayla will be paying the same interest rate.

Determine P .

Solution:

Option a:

$$10,000(1+i)^{10} = 26,017.40 ==> i = \left(\frac{26,017.40}{10,000}\right)^{1/10} - 1 = 0.100339$$

Option b:

For this option since payments are monthly, we need $\frac{i^{(12)}}{12}$.

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = 1+i = 1.100339 ==> \frac{i^{(12)}}{12} = (1.100339)^{1/12} - 1 = 0.008$$

$$Pa_{\overline{120|}} = 10,0000 ==> P\left(\frac{1-(1.008)^{-120}}{0.008}\right) = 10,000 ==> P = \frac{10,0000}{\frac{1-(1.008)^{-120}}{0.008}} = 129.95$$

Alex wants to invest for his retirement. Today is his 22nd birthday.

He will make a payment of 10,000 on each birthday beginning with his 30th birthday. His last payment will be on his 64th birthday.

Alex earns an annual effective interest rate of 6%.

Calculate the amount that Alex will have at the age 65.

Solution:

First, we note that the number of payments is 35. Second, we note that we want the accumulated value at age 65 which is one period after the last payment at age 64.

$$Answer = (10,000)\ddot{s}_{\overline{35}|} = (10,000)\left(\frac{(1.06)^{35}-1}{0.06}\right)(1.06) = 1,181,208.67$$

Alan buys a house for 300,000. He finances the entire amount with a mortgage. The mortgage requires monthly payments for 30 years. The interest rate on the mortgage is an annual effective interest rate of 9%.

Calculate Alan's monthly payment.

Solution:

Payments are monthly so we need the monthly effective interest rate but we are given the annual effective interest rate.

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = 1.09 \Longrightarrow \frac{i^{(12)}}{12} = (1.09)^{1/12} - 1 = 0.007207323$$

$$Qa_{\overline{360|}} = 300,000 \Longrightarrow Q = \frac{300,000}{\left(\frac{1-(1.007207323)^{-360}}{0.007207323}\right)} = 2338.45$$

Kylie invests P at the beginning of each year into an account which earns an annual effective interest rate of 7.2%.

Kylie's balance at the end of 13 years is 100,000.

Determine P .

Solution:

$$P\ddot{s}_{\overline{13}|} = 100,000$$

$$P = \frac{100,000}{\ddot{s}_{\overline{13}|}} = \frac{100,000}{\left(\frac{(1.072)^{13} - 1}{0.072}\right)(1.072)} = 4571.91$$

You are given that $1000a_{\overline{n}|} = 12,676.16$ and $1000s_{\overline{n}|} = 45,068.90$.

Both annuity values are calculated using the same interest rate.

Determine n.

Solution:

$$\frac{1}{a_{\overline{n}|}} = \frac{1}{s_{\overline{n}|}} + i$$

$$1000a_{\overline{n}|} = 12,676.16 \Longrightarrow a_{\overline{n}|} = 12.67616 \text{ and } 1000s_{\overline{n}|} = 45,068.90 \Longrightarrow s_{\overline{n}|} = 45.06890$$

$$\frac{1}{12.67616} = \frac{1}{45.06890} + i \Longrightarrow i = 0.0567$$

If you do not remember the formula above, you still find i using algebra.

$$\mathrm{a}_{\overline{n}|}(1+i)^n = s_{\overline{n}|} \Longrightarrow 12.67616(1+i)^n = 45.06890 \Longrightarrow (1+i)^n = \frac{45.06890}{12.67616} = 3.555406369$$

$$s_{\overline{n}|} = \frac{(1+i)^n - 1}{i} = \frac{3.55506369 - 1}{i} = 45.06890 \Longrightarrow i = \frac{2.55506369}{45.06890} = 0.0567$$

Then

$$\frac{(1.0567)^n - 1}{0.0567} = 45.0689 \Longrightarrow (1.0567)^n = (45.0689)(0.0567) + 1 = 3.55540663$$

$$n = \frac{\ln(3.55540663)}{\ln(1.0567)} = 23$$