Topic: Bond Amortization

A 20 year bond matures for its par value of 50,000. The bond has semi-annual coupons payable at a rate of 6.2% convertible semi-annually. The bond is bought to yield 7.4% convertible semi-annually.

Calculate the write up of discount (which is synonymous with the amount of principal) in the coupon at the end of the 15th year.

Solution:

To find the write up of discount (amount of principal) in the 30th payment (end of 15th year), we need to find the Book Value (OLB) after the 29th payment.

$$i^{(2)} = 0.074 \Rightarrow \frac{i^{(2)}}{2} = \frac{0.074}{2} = 0.037$$

$$r = \frac{0.062}{2} = 0.031$$

After the 29th payment, there are 11 payments left.

$$BV_{29} = Fra_{\overline{11}|} + Cv^{11}$$

$$= (50,000)(0.031)\left(\frac{1-(1.037)^{-11}}{0.037}\right) + (50,000)(1.037)^{-11}$$

$$= 47,328.82$$

$$I_{30} = (47,328.82)(0.037) = 1751.17$$

$$PMT_{30} = Fr = (50,000)(0.031) = 1550$$

$$P_{30} = PMT_{30} - I_{30} = 1550 - 1751.17 = -201.17$$

Teddy owns a bond that pays annual coupons of 500. The book value of the bond right after the 10th coupon is 14,844.42. The book value on the bond right after the 11th coupon is 14,820.92.

Calculate the yield rate on this bond.

Solution:

The easy way:

$$B_{k-1}(1+j) - Fr = B_k \Longrightarrow 14,844.42(1+j) - 500 = 14,820.92$$

$$14,844.42(1+j) = 15,320.92 \Longrightarrow j = \frac{15,320.92}{14,844.42} - 1 = 0.0321$$

Or:

$$Fr = 500 = \mathrm{Pr}_{11} + Int_{11}$$

$$B_{10} - \mathrm{Pr}_{11} = B_{11} \Longrightarrow \mathrm{Pr}_{11} = 14,844.42 - 14,820.92 = 23.50$$

$$Int_{11} = 500 - 23.50 = 476.50$$

$$B_{10} \cdot j = Int_{11} \Longrightarrow (14,844.42)(j) = 476.50 \Longrightarrow j = \frac{476.50}{14,844.42} = 0.0321$$

Danielle buys a 20 year bond issued by Ryan Corporation. The bond has a par value of 10,000 and a maturity value of 12,000. The bond pays semi-annual coupons of 450. Danielle bought the bond to yield $X\%$ convertible semi-annually.

The book value after the 15th coupon is 13,988.06. The amortization of premium in the 16th coupon is 45.72.

Determine X .

Solution:

$Coupon = 450 \qquad BV_{15} = 13{,}988.06 \qquad P_{16} = 45.72 \qquad r = \frac{X}{2}\%$

$I_{16} = Coupon - P_{16} = 450 - 45.72 = 404.28$

$I_{16} = (BV_{15})r = (BV_{15})\left(\frac{X}{2}\right) = 13{,}988.06\left(\frac{X}{2}\right) = 6994.03X$

$X = \frac{404.28}{6994.03} = 0.057803584$

Anunai has a loan that requires three annual payments to repay the loan. The interest rate on the loan is an annual effective interest rate of 6%.

Complete the following amortization table. Show your work for full credit.

Time	Payment	Interest in Payment	Principal in Payment	Outstanding Loan Balance
0	---	---	---	$9000(1.06)^{-1}$ $+7000(1.06)^{-2}$ $+5000(1.06)^{-3}$ $=18{,}918.64$
1	9000	$(18{,}918.64)(0.06)$ $=1135.12$	$9000 - 1135.12$ $=7864.88$	$18{,}918.64 - 7864.88$ $=11{,}053.76$
2	7000	$(11{,}053.76)(0.06)$ $=663.23$	$7000 - 663.23$ $=6336.77$	$11{,}053.76 - 6336.77$ $=4716.99$
3	5000	$(4716.99)(0.06)$ $=283.02$	$5000 - 283.02$ $=4716.98$	$4716.99 - 4716.98$ $=0.01$

A 25 year bond has annual coupons of 1000 and a maturity value of C .

The interest in the 10th coupon is 978.87. The interest in the 20th coupon is 957.45.

The book value at the end of the 22 year after the coupon is paid is 13,069.15.

Determine C .

Solution:

$$P_{10} = 1000 - 978.87 = 21.13$$

$$P_{20} = 1000 - 957.45 = 42.55$$

$$P_{10}(1+j)^{10} = P_{20} \Longrightarrow 21.13(1+i)^{10} = 42.55 \Longrightarrow i = \left(\frac{42.55}{21.13}\right) - 1 = 0.0725$$

Book Value is the present value of future cash flows.

$$B_{22} = 13,069.15 = 1000a_{\overline{3}|} + C(1.0725)^{-3} \Longrightarrow C = \frac{13,069.15 - 1000\left(\frac{1-(1.0725)^{-3}}{0.0725}\right)}{(1.0725)^{-3}} = 12,900$$

Kanishk can purchase either of the following two bonds:

a. Bond A has a par value of 25,000 and semi-annual coupons. The bond sells for 30,000. The coupon rate is 6% convertible semi-annually. The amount of principal in the first coupon is 71.70.

b. Bond B is a 20 year zero coupon bond. This bond also has a price of 30,000.Bond A and Bond B have the same yield rate.

Calculate the maturity value of Bond B.

Solution:

$BV_0 = \text{Price} = 30,000$

$Coupon = (25,000)(0.06/2) = 750$

$I_1 = Coupon - P_1 = 750 - 71.70 = 678.30$ but $I_1 = (BV_0)r = (30,000)(r)$

$r = \dfrac{678.30}{30,000} = 0.02261$

Price of Bond B = $(\text{Maturity Value})(1+r)^{-20(2)}$ since r is for a six month period.

$\text{Maturity Value} = (30,000)(1.02261)^{40} = 73,370.70$

Tom purchases a 2 year bond which matures for 20,000. The bond has semi-annual coupons. The coupons are not level. The first two coupons are each equal to 1000. The second two coupons are each equal to 2000.

The bond is bought to yield 13% convertible semi-annually.

Complete the following amortization table for Tom's bond. Show formulas if you want full credit.

Time k	Coupon	Interest in Coupon	Principal in Coupon	Book Value
0	---	---	---	Present Value of Cash Flows = $1000v+1000v^2+2000v^3+(2000+20000)v^4$ =20,577.43
1	1000	(20,577.43)(0.065) = 1337.53	1000 – 1337.53 = -337.53	20,577.43 – (-337.53) =20,914.96
2	1000	(20,914.96)(0.065) = 1359.79	1000 – 1359.47 = -359.47	20,914.96 – (-359.47) =21,277.43
3	2000	(21,277.43)(0.065) = 1382.84	2000 – 1382.84 = 617.16	21,277.43 – 617.16 = 20,657.27
4	2000	(20,657.27)(0.065) = 1342.72	2000 – 1342.72 = 657.28	20,657.27 – 657.28 = 20,000