Topic: Bonds

A 20 year bond has non-level coupons which are paid semi-annually. The bond matures for 100,000. Each coupon in the first year is 300. Each coupon in the second year is 600. Each coupon in the third year is 900. The coupons continue to increase in the same pattern until each coupon in the 20 year is 6000.

The bond is bought to yield 6% compounded semi-annually.

Determine the price of the bond.

Solution:

The price is the present value of cash flows. We have to use the Formula That Does Not Follow The Rules since the payments are level during the year but increase year to year.

$$i^{(2)} = 0.06 \Rightarrow \frac{i^{(2)}}{2} = \frac{0.06}{2} = 0.03$$

$$i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1 = (1.03)^2 - 1 = 0.0609$$

$$PV = 300\left(\frac{\ddot{a}_{\overline{20|}} - 20(1+i)^{-20}}{\frac{i^{(2)}}{2}}\right) + 100,000\left(1 + \frac{i^{(2)}}{2}\right)^{-40}$$

$$= 300\left(\frac{\left[\frac{1-(1.0609)^{-20}}{0.0609}(1.0609)\right] - 20(1.0609)^{-20}}{0.03}\right) + 100,000(1.03)^{-40}$$

$$= 59,488.94 + 30,655.68$$

$$= 90,144.62$$

A 20 year bond has a maturity value of C and a par value of $0.9C$. The bond pays semi-annual coupons of 250. The bond is bought for a price of 10,367.59 to yield 5% convertible semi-annually.

Determine if this bond is purchased at a discount or a premium and state the amount of discount or premium.

Solution:

$$P = Fra_{\overline{40}|} + Cv^{40}$$

$$10,367.59 = (250)\left(\frac{1-(1.025)^{-40}}{0.025}\right) + (C)(1.025)^{-40}$$

$$10,367.59 = 6275.69 + (0.372430624)(C)$$

$$C = \frac{10,367.59 - 6275.69}{0.372430624} = 10,987.00$$

Since $C > P \Rightarrow Discount = 10,987.00 - 10,367.59 = 619.41$

Mary buys a 20 year bond with semi-annual coupons at a rate of 6% compounded semi-annually. The bond has a par value of F and matures for $F+100$. The price of the bond is 1029.78 using a yield rate of 5.8% compounded semi-annually.

Determine the amount of the premium or the discount in the sale of this bond. Be sure to state whether it is a premium or discount.

Solution:

$$P = Fra_{\overline{40}|} + Cv^{40}$$

$$1029.78 = F(0.03)\left(\frac{1-(1.029)^{-40}}{0.029}\right) + (F+100)(1.029)^{-40}$$

$$1029.78 = (0.704790867)F + (0.318702162)F + 31.87$$

$$F = \frac{1029.78 - 31.87}{1.023493029} = 975.00$$

$$C = 975.00 + 100.00 = 1075.00$$

$$\text{Since } C > P \Longrightarrow Discount = 1075.00 - 1029.78 = 45.22$$

A 20 year bond has a par value of 1000 and a maturity value of 1300. The semi-annual coupon rate for the bond is 7.5% convertible semi-annually. The bond is purchased to yield 9% convertible semi-annually.

Calculate the principal in the coupon paid at the end of the 12^{th} year.

Solution:

This is a question that can be done on your calculator.

$$\boxed{FV} \leftarrow 1300; \boxed{PMT} \leftarrow (1000)(0.075/2) = 37.50; \boxed{N} \leftarrow (20)(2) = 40; \boxed{I/Y} \leftarrow 9/2 = 4.5$$

$$\boxed{CPT}\boxed{PV}\boxed{2nd}\boxed{Amort}\boxed{P1} \leftarrow \boxed{P2} \leftarrow (12)(2) = 24\boxed{\downarrow}\boxed{\downarrow}\boxed{PRN} \rightarrow -9.94$$

Xinyue has the choice of the following two bonds:

a. Bond A is a 10 year par value bond with a maturity value of 10,000. The bond has a coupon rate of 8% convertible semi-annually.

b. Bond B is a 10 year bond with a par value of F and a maturity value of $F+200$. The bond has a coupon rate of 7% convertible semi-annually.

Both bonds sell for a price of P to yield 7.5% convertible semi-annually. As a result, Bond B is purchased at a discount.

Determine the amount of the discount on Bond B.

Solution:

We use Part a to find the price P.

$$P=(10,000)(0.08/2)\left(\frac{1-(1.0375)^{-20}}{0.0375}\right)+(10,000)(1.0375)^{-20}=10,347.41$$

Part b

$$P=0.035F\left(\frac{1-(1.0375)^{-20}}{0.0375}\right)+(F+200)(1.0375)^{-20}=10,347.41$$

$$\Longrightarrow (0.486367147)F+(0.478892342)F+95.77846841=10,347.41$$

$$F=\frac{10,347.41-95.78}{0.486367147+0.478892342}=10,620.59$$

$$C=F+200=10,820.59$$

$$Discount=C-P=10,820.59-10,347.41=473.18$$

Rahul buys a 20 year bond with semi-annual coupons. The maturity value of the bond is 100,000.

The coupons increase. The first coupon is 200. The second coupon is 400. The third coupon is 600. The coupons continue to increase in the same pattern until the last coupon of 8000 is paid.

The bond is bought to yield 10% convertible semi-annually.

Determine the price of the bond.

Solution:

Price = PV of Cash Flows

$$= 200a_{\overline{40}|} + \frac{200}{0.05}\left(a_{\overline{40}|} - 40(1.05)^{-40}\right) + 100,000(1.05)^{-40}$$

$$= 200\left(\frac{1-(1.05)^{-40}}{0.05}\right) + \frac{200}{0.05}\left(\left(\frac{1-(1.05)^{-40}}{0.05}\right) - 46(1.05)^{-40}\right) + 100,000(1.05)^{-40}$$

$$= 63,545.42$$

Each of the following two bonds has a price of 50,000 and the same yield rate:

a. A zero coupon bond that matures for 100,000 at the end of 10 years.

b. A 10 year bond with a maturity value of 80,000 and semi-annual coupons of X .

Determine X .

Solution:

Bond a.

$$50,000 = 100,000(1+i)^{-10} \Longrightarrow 0.5 = (1+i)^{-10} \Longrightarrow 2 = (1+i)^{10} \Longrightarrow i = (2)^{0.1} - 1 = 0.0717735$$

Bond b.

We need $\frac{i^{(2)}}{2}$ since coupons are semi-annually.

$$\left(1+\frac{i^{(2)}}{2}\right)^2 = 1+i = 1.0717735 \Longrightarrow \frac{i^{(2)}}{2} = (1.0717735)^{0.5} - 1 = 0.035264924$$

$$50,000 = Xa_{\overline{20}|} + 80,000(1.035264924)^{-20} = X\left(\frac{1-(1.035264924)^{-20}}{0.035264924}\right) + 80,000(1.035264924)^{-20}$$

$$X = \frac{50,000 - 80,000(1.035264924)^{-20}}{\left(\frac{1-(1.035264924)^{-20}}{0.035264924}\right)} = 705.30$$

A 20 year bond is sold at a discount of 380. The bond has a par value of F and a maturity of $F+400$. The bond pays semi-annual coupons at a rate of 6% compounded semi-annually.

The bond is purchased to yield 6.5% compounded semi-annually.

Determine the price of the bond.

Solution:

$$C - P = Discount = 380 ==> P + 380 = C$$

$$C = F + 400 ==> F + 400 = P + 380 ==> P = F + 20$$

$$P = Fra_{\overline{40}|} + Cv^{40} ==> F + 20 = F(0.03)\left(\frac{1-(1.0325)^{-40}}{0.0325}\right) + (F + 400)(1.0325)^{-40}$$

$$F + 20 = F(0.666253) + 0.27822592F + 111.2903679$$

$$F(1 - 0.666253 - 0.27822592) = 111.2903679 - 20$$

$$F = 1644.25$$

$$P = F + 20 = 1664.25$$

A 20 year bond has a maturity value of 100,000 and semi-annual coupons that are not level. The two coupons paid during the first year are 500 each. The two coupons paid during the second year are 1000 each. The coupons continue increasing in the same pattern until the two coupons paid during the 20th year are 10,000 each.

The bond is purchased to yield 8% convertible semi-annually.

Determine the price of this bond.

Solution:

The price of the bond is the PV of future cash flows. The coupons follow a pattern that will require us to use the formula that does not follow the rules so we will need both $\frac{i^{(2)}}{2}$ and i. We are given $i^{(2)} = 0.08$ so $\frac{i^{(2)}}{2} = 0.04$ and $i = (1.04)^2 - 1 = 0.0816.$

$$\text{Price} = 500\left(\frac{\ddot{a}_{\overline{20}|0.0816} - 20(1.0816)^{-20}}{0.04} \right) + 100,000(1.04)^{-40}$$

$$= 500\left(\frac{\frac{1-(1.0816)^{-20}}{0.0816}(1.0816) - 20(1.0816)^{-20}}{0.04} \right) + 100,000(1.04)^{-40} = 99,932.28$$

Connor buys a 12 year bond with a par value of F. The bond matures for $F+500$. The bond has semi-annual coupons paid at a rate of 7% convertible semi-annually.

At yield rate of 9% compounded semi-annually, bond is bought at a discount of 400.

Determine F.

Solution:

$$C - P = 400 ==> P = C - 400$$

$$C = F + 500 ==> P = F + 500 - 400 = F + 100$$

$$P = Fra_{\overline{24}|} + Cv^{24}$$

$$F + 100 = (F)(0.07/2)\left(\frac{1-(1.045)^{-24}}{0.045}\right) + (F+500)(1.045)^{-24}$$

$$F = \frac{(500)(1.045)^{-24} - 100}{1-(0.07/2)\left(\frac{1-(1.045)^{-24}}{0.045}\right) - (1.045)^{-24}} = \frac{73.85173676}{0.144954784} = 509.48$$

A 10 year bond pays semi-annual coupons that are increasing. The first coupon is 500. The second coupon is 600. The third coupon is 700. The coupons continue to increase in the same pattern. The bond has a maturity value of 13,000.

Calculate the price of the bond to yield 10% convertible semi-annually.

Solution:

The price is the present value of cash flows. Since each coupon is increasing, we must use the P&Q formula.

$$PV = 500a_{\overline{20}|} + \frac{100}{0.05}\left(a_{\overline{20}|} - 20(1.05)^{-20}\right) + (13,000)(1.05)^{-20}$$

$$= 500\left(\frac{1-(1.05)^{-20}}{0.05}\right) + \frac{100}{0.05}\left(\frac{1-(1.05)^{-20}}{0.05} - 20(1.05)^{-20}\right) + (13,000)(1.05)^{-20}$$

$$= 20,979.51$$