Topic: Bottom Line Approach

Alberto, Brian, and Vanessa enter into a financial agreement. Alberto agrees to pay Brian 10,000 today. Alberto will also pay 20,000 to Vanessa today.

At the end of one year, Brian will pay Vanessa 7,000.

At the end of two years, Vanessa will pay 20,000 to Alberto.

At the end of four years, Vanessa will pay 14,000 to Alberto. Additionally, at the end of four years, Brian will pay 5,723 to Alberto.

Finally, at the end of 5 years, Vanessa pays X to Brian.

Over the five year period, the annual effective interest rate paid or received by each party is the same.

Determine X .

Solution:

Alberto's cashflows are -30,000 at time 0, 20,000 at time 2, 19,723 at time 4.

His equation of value is:

$$30,000(1+i)^4 = 20,000(1+i)^2 + 19,723$$

Let $x = (1+i)^2$

$$30,000x^2 = 20,000x + 19,723 \implies 30,000x^2 - 20,000x - 19,723 = 0$$

$$x = \frac{-(-20,000) \pm \sqrt{(-20,000)^2 - 4(30,000)(-19,723)}}{2(30,000)} = 1.21$$

$$x = (1+i)^2 = 1.21 \implies i = (1.21)^{\frac{1}{2}} - 1 = 0.1$$

(SOLUTION CONTINUED BELOW)

Brian's cashflows are 10,000 at time 0, -7,000 at time 1, -5,723 at time 4, X at time 5.

$$10,000(1.1)^5 + X = 7,000(1.1)^4 + 5,723(1.1)$$

$$X = 7,000(1.1)^4 + 5,723(1.1) - 10,000(1.1)^5 = 438.90$$

OR

Vanessa's cashflows are 20,000 at time 0, 7,000 at time 1, -20,000 at time 2, -14,000 at time 4, -X at time 5.

$$20,000(1.1)^5 + 7,000(1.1)^4 = 20,000(1.1)^3 - 14,000(1.1) + X$$

$$X = 20,000(1.1)^5 + 7,000(1.1)^4 - 20,000(1.1)^3 - 14,000(1.1) = 438.90$$

$$X = 438.90$$

Reagan, Danial and Claire enter into a financial arrangement. Under this arrangement, Reagan will pay Danial 10,000 today. Danial will pay Regan P at the end of 3 years. Danial will also pay Claire 6000 at the end of 4 years. Claire will pay Reagan P at the end of 8 years.

Using the bottom line approach, Reagan's annual effective interest rate on her investment is 6%.

Determine P.

Solution:

Reagan's cashflows are $-10,000$ at time 0, P at time 3 and P at time 8.

Her equation of value is

$$10{,}000(1.06)^8 = P(1.06)^5 + P$$

$$P = \frac{10{,}000(1.06)^8}{(1.06)^5 + 1} = 6816.49$$

Anthony, Walker, and Lyndi enter into a business arrangement. Under this arrangement, Anthony will pay Walker 1000 at time 0 and will pay Lyndi 2000 at time 2. Also, Walker will pay Lyndi 1200 at time 2. Finally, Lyndi will pay Anthony 3600 at time 4.

Using the bottom line approach, determine the annual effective interest rate being earned by Anthony is this arrangement.

Solution:

Anthony's cash flows are as follows:

Time 0 ==> −1000

Time 2 ==> −2000

Time 4 ==> +3600

Using the calculator:

$\boxed{CF0 \leftarrow -1000}; \boxed{C01 \leftarrow 0} \boxed{C02 \leftarrow -2000} \boxed{C03 \leftarrow 0} \boxed{C04 \leftarrow 3600}$

$\boxed{IRR} \boxed{CPT} \rightarrow 6.9935\%$

David, Max, and Andrew enter into a financial agreement. Under this agreement, David will pay Max 1000 today. David will also pay Andrew 800 at the end of one year. Max will pay Andrew 1100 at the end of one year. Finally, Andrew will pay David 2000 at the end of two years.

Calculate the yield rate that David will receive under this financial arrangement.

Solution:

David's cash flows are a payment of 1000 at time 0, a payment of 800 at time 1, and receipt of a payment of 2000 at time 2.

To find the yield rate, we can use our calculator:

$$CF0 \leftarrow -1000; C01 \leftarrow -800; C02 \leftarrow 2000; \boxed{IRR}; \boxed{CPT} \Longrightarrow 6.9694\%$$

Or :

$$1000(1+i)^2 + 800(1+i) = 2000 \qquad \text{Let } x = 1+i$$

$$1000x^2 + 800x - 2000 = 0$$

$$1+i = x = \frac{-800 \pm \sqrt{(800)^2 - (4)(1000)(-2000)}}{(2)(1000)} = 1.069694 \Longrightarrow i = 6.9694\%$$

Sam, Moses, and Jacquelyn enter into a business agreement.

Under this agreement, Sam will pay Moses 1000 today. He will also pay Jacquelyn 2000 at the end of two years.

Further, Moses will pay Jacquelyn 500 at the end of four year and he will pay Sam 700 at the end of two years.

Finally, Jacquelyn will pay Sam 3000 at the end of six years.

Calculate Jacquelyn's annual effective interest rate on this business agreement.

Solution:

Jacquelyn's payments are:
Receives 2000 at time 2 from Sam
Receives 500 at time 4 from Moses
Pays 3000 at time 6 to Sam

We will use time 6 as our point of value.

$2000(1+i)^4 + 500(1+i)^2 = 3000 \qquad \text{Let } x = (1+i)^2$

$$2000x^2 + 500x - 3000 = 0 \Longrightarrow x = \frac{-500 \pm \sqrt{500^2 - (4)(2000)(-3000)}}{(2)(2000)} = 1.106107225$$

$$x = (1+i)^2 = 1.106107225 \Longrightarrow i = (1.106107225)^{0.5} - 1 = 0.051716324$$

Or we can use our calculator and starting with the first payment
$CF0 = -2000; C01 = 0; C02 = -500; C03 = 0; C04 = 3000$

$\boxed{IRR}\boxed{CPT} \rightarrow 5.1716324\%$

Giacomo, Yuchen, and Cai enter into a financial agreement. Under the agreement, Giacomo pays Yuchen 10,000 now. Additionally, he pays Cai 12,000 at the end of N years.

Yuchen pays Cai 4000 at the end of two years and pays 9000 to Giacomo at the end of 2N years.

Cai also pays Giacomo 20,000 at the end of 2N years.

Giocomo realizes an annual effective return of 12% on this financial arrangement.

Determine N. (Note: N is not an integer.)

Solution:

First you must find Giacomo's payments:

Giacomo will pay 10,000 at time 0 and 12,000 at time N. He will receive 20,000 plus 9000 at time 2N.

Now set up our equation of value:

$$-10,000(1.12)^{2N} - 12,000(1.12)^{N} + 29,000 = 0$$

Let $x = (1.12)^{N}$

$$-10,000(x)^{2} - 12,000x + 29,000 = 0$$

$$\Longrightarrow x = \frac{-(-12,000) \pm \sqrt{(-12,000)^{2} - (4)(-10,000)(29,000)}}{(2)(-10,000)} = 1.205547$$

$$1.205547 = (1.12)^{N} \Longrightarrow N = \frac{\ln(1.205547)}{\ln(1.12)} = 1.64948$$