Topic: Convexity

Christopher has agreed to pay Anthony an annuity due with four payments of 1000. The payments are at the beginning of each 6 months period for two years.

Calculate the Macaulay Convexity of this annuity at an annual effective interest rate of 6%.

Solution:

Since payments are made semiannually, we need $\frac{i^{(2)}}{2}$ and we have i .

$$\frac{i^{(2)}}{2} = (1+i)^{\frac{1}{2}} - 1 = (1.06)^{\frac{1}{2}} - 1 = 0.029563$$

$$MacCon = \frac{\Sigma C_t (t^2) v^t}{\Sigma C_t v^t}$$

$$= \frac{1{,}000(0)v^0 + 1{,}000(0.5)^2 v^1 + 1{,}000(1)^2 v^2 + 1{,}000(1.5)^2 v^3}{1{,}000\, \ddot{a}_{\overline{4}|0.029563}}$$

$$= \frac{1{,}000(0.5)^2(1.06)^{-0.5} + 1{,}000(1)^2(1.06)^{-1} + 1{,}000(1.5)^2(1.06)^{-1.5}}{1{,}000\, \ddot{a}_{\overline{4}|0.029563}}$$

$$= \frac{1{,}000(0.5)^2(1.06)^{-0.5} + 1{,}000(1)^2(1.06)^{-1} + 1{,}000(1.5)^2(1.06)^{-1.5}}{1{,}000\left(\frac{1-(1.029563)^{-4}}{0.029563}\right)(1.029563)}$$

$$= 0.84779908$$

Shikun is the recipient of an annuity that will pay the following:

i. 100,000 today;
ii. 200,000 at the end of two years; and
iii. 300,000 at the end of four years.

Calculate the Macaulay Convexity of Shikun's payments at an interest rate of 4%.

Solution:

$$MacCon = \frac{\sum C_t \cdot t^2 \cdot v^t}{\sum C_t \cdot v^t} = \frac{(100,000)(0^2)(v^0) + (200,000)(2^2)v^2 + (300,000)(4^2)v^4}{100,000 + 200,000v^2 + 300,000v^4}$$

$$= \frac{4,842,705.087}{541,352.4999} = 8.945567053$$

Tomas has agreed to pay the following payments to Taylen:

a. 100,000 at the end of one year;

b. 250,000 at the end of two years; and

c. 400,000 at the end of four years.

Calculate the Modified Convexity of these payments at an interest rate of 8%.

Solution:

$$ModCon = v^2 \frac{\sum C_t (t)(t+1)v^t}{\sum C_t v^t}$$

$$= (1.08)^{-2} \left(\frac{(100,000)(1)(2)(1.08)^{-1} + (250,000)(2)(3)(1.08)^{-2} + (400,000)(4)(5)(1.08)^{-4}}{(100,000)(1.08)^{-1} + (250,000)(1.08)^{-2} + (400,000)(1.08)^{-4}} \right)$$

$$= 10.488$$

A three year bond has a maturity value of 10,000 and annual coupons of 600.

Calculate the Modified Convexity of this bond at an annual effective interest rate of 6%.

Solution:

$$ModCon = v^2 \frac{\sum C_t (t)(t+1) v^t}{\sum C_t v^t}$$

$$= (1.06)^{-2} \frac{(600)(1)(2)(1.06)^{-1} + (600)(2)(3)(1.06)^{-2} + (10,600)(3)(4)(1.06)^{-3}}{(600)(1.06)^{-1} + (600)(1.06)^{-2} + (10,600)(1.06)^{-3}}$$

$$= 9.8910$$

Ashley just got fired by Huljack LTD. Huljack has agreed to make the following payments to Ashley as a severance package:

a. Payment of 100,000 today;

b. Payment of 200,000 at the end of two years; and

c. Payment of 400,000 at the end of four years.

Calculate the Modified Convexity of Ashley's payments at an annual effective interest rate of 8%.

Solution:

$$ModCon = v^2 \frac{\sum C_t (t)(t+1)v^t}{\sum C_t v^t} =$$

$$(1.08)^{-2} \frac{(200,000)(2)(3)(1.08)^{-2} + (400,000)(4)(5)(1.08)^{-4}}{100,000 + (200,000)(1.08)^{-2} + (400,000)(1.08)^{-4}} = 10.475$$

Beau is receiving the following payments from a trust fund:

a. 200,000 at time 2
b. 400,000 at time 10
c. 800,000 at time 16

Calculate the Macaulay Convexity of these payments at an interest rate of 7%.

Solution:

$$MacCon = \frac{\sum C_t (t^2) v^t}{\sum C_t v^t}$$

$$= \frac{(200,000)(2^2)(1.07)^{-2} + (400,000)(10^2)(1.07)^{-10} + (800,000)(16^2)(1.07)^{-16}}{(200,000)(1.07)^{-2} + (400,000)(1.07)^{-10} + (800,000)(1.07)^{-16}}$$

$$= 139.80$$

The Purdue Insurance Company has agreed to pay Summer 100,000 at the end of 2 years and 200,000 at the end of 3.5 years.

Calculate the Macaulay Convexity of Summer's payments at an annual effective interest rate of 9%.

Solution;

$$MacCon\frac{\sum C_t(t^2)v^t}{\sum C_t v^t}$$

$$=\frac{(100,000)(2^2)(1.09)^{-2}+(200,000)(3.5^2)(1.09)^{-3.5}}{100,000(1.09)^{-2}+200,000(1.09)^{-3.5}}$$

$$=9.258$$

A three year bond has an annual coupon of 40 and a maturity value of 1100.

Calculate the Modified Convexity for this bond at an annual effective interest rate of 6.5%.

Solution:

$$ModCon = \frac{v^2 \sum C_t (t)(t+1) v^t}{\sum C_t v^t}$$

$$= (1.065)^{-2} \frac{40(1)(2)(1.065)^{-1} + 40(2)(3)(1.065)^{-2} + 1140(3)(4)(1.065)^{-3}}{40 a_{\overline{3}|} + 1100(1.065)^{-3}}$$

$$= 10.07065$$

A three year bond with annual coupons of 300 matures for 4000.

Calculate the Macaulay Convexity of this bond at an annual effective rate of 5%.

Solution:

$$MacCon = \frac{\sum C_t (t^2) v^t}{\sum C_t v^t}$$

$$= \frac{300(1^2)(1.05)^{-1} + 300(2^2)(1.05)^{-2} + 4300(3^2)(1.05)^{-3}}{300a_{\overline{3|}} + 4000(1.05)^{-3}}$$

$$= \frac{34,804.66472}{4272.324803} = 8.14654$$