Topic: Determinants of Interest Rates

. Nick Bank makes 3 ½ year loans to college seniors. Nick wants to earn an annual rate of 3.2% compounded continuously in order defer consumption. Additionally, Nick expects inflation to be an annual rate of 2.9% compounded continuously during the loan. Finally, since the inflation rate over the time of the loan is unknown, Nick wants to earn an additional annual rate of 0.4% compounded continuously as compensation of the uncertainty of the inflation rate.

College seniors as a group have a default rate of 8%. For those that default, the recovery rate is 40%. Nick includes default costs in his total interest rate charged.

Sneha is a college senior and borrows 28,000 from Nick Bank. At the end of 3 ½ years, Sneha repays the loan.

Determine the amount that Sneha will pay at the end of 3 ½ years to repay the loan.

Solution:

$$R = 0.032 + 0.029 + 0.004 + s = 0.065 + s$$

$$28,000e^{0.065(3.5)} = 28,000(0.92)e^{(0.065+s)(3.5)} + 28,000(0.08)(0.40)e^{(0.065+s)(3.5)}$$

$$e^{0.065(3.5)} = (0.92)e^{(0.065+s)(3.5)} + (0.08)(0.40)e^{(0.065+s)(3.5)} = 0.952e^{(0.065+s)(3.5)}$$

$$\frac{e^{0.065(3.5)}}{0.952} = e^{(0.065+s)(3.5)} \implies \ln\left[\frac{e^{0.065(3.5)}}{0.952}\right] = \ln\left[e^{(0.065+s)(3.5)}\right]$$

$$0.276690244 = (0.065 + s)(3.5) = 0.2275 + 3.5s$$

$$s = \frac{0.276690244 - 0.2275}{3.5} = 0.014054355$$

$$R = 0.065 + 0.014054355 = 0.079054355$$

$$Amount = 28,000e^{(0.079054355)(3.5)} = 36,925.22$$

Huber Bank makes three year loans. Huber wants to receive an annual rate of 3.5% compounded continuously to defer consumption. Huber believes that the annual rate of inflation for the next three years will be 1.8% compounded continuously. Additionally, Huber charges an annual rate of 30 basis points compounded continuously for the risk that inflation could exceed expectations.

Huber believes that 4% of the loans will default and the recovery rate on the defaulted loans will be 60%.

Calculate the total annual interest rate compounded continuously that Huber will charge on these three year loans.

Solution:

$$\text{Rate without Defaults} = 0.035 + 0.018 + 0.003 = 0.056$$

$$\text{Rate with Defaults} = 0.056 + \delta_s$$

$$e^{(0.056)(3)} = (1-0.04)e^{(0.056+\delta_s)(3)} + (0.04)(0.6)e^{(0.056+\delta_s)(3)} = 0.984e^{(0.056+\delta_s)(3)}$$

$$\frac{e^{(0.056)(3)}}{0.984} = e^{(0.056+\delta_s)(3)} \implies (0.056+\delta_s)(3) = \ln\left[\frac{e^{(0.056)(3)}}{0.984}\right] = 0.184129382$$

$$\delta_s = \frac{0.184129382}{3} - 0.056 = 0.005376461$$

$$\text{Total Interest Rate} = 0.056 + 0.005376461 = 0.0613765$$

The US Government issues four year inflation protected loans. The real interest rate compounded continuously during the four years is 2.5% annually. The US Government is considered a risk free borrower so there is no default charge.

The annual inflation rate compounded continuously for the first two years of the loan is 4%. The annual inflation rate compounded continuously for the last two years of the loan is $i_a\%$.

Spencer borrows 100,000 from the US Government and repays 122,556.26 at the end of four years.

Determine $i_a\%$.

Solution:

$$100,000e^{(0.025)(4)}e^{(0.04)(2)}e^{(i_a)(2)} = 122,556.26$$

$$e^{(i_a)(2)} = \frac{1.2255626}{e^{(0.025)(4)}e^{(0.04)(2)}} = 1.023675932$$

$$(i_a)(2) = \ln(1.023675932) \Longrightarrow i_a = 1.17\%$$

Rodgers Bank makes five year loans to college students. Rodgers wants to receive an annual rate of 5% compounded continuously to compensate for deferred consumption. Additionally, Rodgers expects that inflation will occur at an annual rate of 3% compounded continuously over the next five years. However, since the inflation rate could be higher, Rodgers would like to receive an annual rate of 0.5% compounded continuously as compensation for the inflation risk.

Additionally, Rogers expects 10% of the loans to default with a loan recovery rate of 48%. Rogers adds a charge for defaults to the other components of the interest rates to compensate for the expected defaults.

Determine the annual rate compounded continuously that Rodgers should charge for defaults. Your answer should be accurate to five decimal places.

Solution:

$$\text{Rate without Defaults} = 0.05 + 0.03 + 0.005 = 0.085$$

$$\text{Rate with Defaults} = 0.05 + 0.03 + 0.005 + \delta_s = 0.085 + \delta_s$$

Total Cash Received without defaults must equal total cash received with defaults.

$$e^{(0.085)(5)} = 0.9e^{(0.085+\delta_s)(5)} + (0.1)(0.48)e^{(0.085+\delta_s)(5)} = (0.948)e^{(0.085+\delta_s)(5)}$$

$$\frac{e^{(0.085)(5)}}{e^{(0.085)(5)}} = \frac{(0.948)e^{(0.085+\delta_s)(5)}}{e^{(0.085)(5)}} ==> 1 = (0.948)e^{(\delta_s)(5)} ==> e^{(\delta_s)(5)} = \frac{1}{0.948}$$

$$(\delta_s)(5) = \ln\left(\frac{1}{0.948}\right) ==> \delta_s = 0.01068$$

Wang National Bank makes five year loans for college students. Wang wants to receive an annual interest rate 3.5% compounded continuously without taking into account defaults and inflation.

Wang expects that inflation for the next five years will be at an annual rate of 2.5% compounded continuously. However, since this inflation assumption is only an expectation, inflation could be higher or lower. Therefore, Wang also charges an annual rate of 0.4% compounded continuously on all loans to compensate for the uncertainty of the inflation expectation.

College students have a high default rate. Wang believes that 5% of the students will default on the loan at the end of five years. Wang also believes that the bank will be able to recover 45% of the amount owed on defaults.

Calculate the credit spread that Wang needs to charge as an annual rate compounded continuously.

Solution:

$$R^{WithoutDefaults} = 0.035 + 0.025 + 0.004 = 0.064$$

$$R^{WithDefautls} = 0.064 + s$$

$$e^{0.064(5)} = (0.95)e^{(0.064+s)(5)} + (0.05)(0.45)e^{(0.064+s)(5)}$$

$$e^{0.064(5)} = (0.9725)e^{(0.064+s)(5)} \Longrightarrow \frac{e^{(0.064+s)(5)}}{e^{0.064(5)}} = \frac{1}{0.9725} \Longrightarrow e^{5s} = \frac{1}{0.9725}$$

$$5s = \ln\left[\frac{1}{0.9725}\right] \Longrightarrow s = \frac{\ln\left[\frac{1}{0.9725}\right]}{5} = 0.00558$$

Sue lends 100,000 to Nathan. The loan is for four years and includes inflation protection. Nathan will repay an annual interest rate of 5.2% compounded continuously plus the rate of inflation. The 5.2% compounded continuously already reflects the cost of inflation protection and the cost of defaults.

The rate of inflation in the first year was 2.3% compounded continuously. The rate of inflation in the second and third years was $x\%$ compounded continuously. The rate of inflation in the last year of the loan was 3.5% compounded continuously.

At the end of four years, Nathan repays the loan with a payment of 143,000.

Calculate x.

Solution:

$$(100,000)e^{0.052+0.023+0.052+x+0.052+x+0.052+0.035} = 143,000$$

$$e^{0.266+2x} = 1.43$$

$$0.266 + 2x = \ln(1.43) \Longrightarrow x = \frac{\ln(1.43) - 0.266}{2} = 0.04584$$