Topic: Discount Rate and Function

Michelle invests 10,000 for 10 years. During the first 3 years, Michelle earns a simple interest rate of 10%. During the next 5 years, Michelle earns a compound interest rate of 8%. During the last two years, Michelle earns a rate of interest equivalent to an annual effective discount rate of 6%.

Determine the amount that Michelle will have at the end of 10 years.

Solution:

For the first three years, simple interest of 10%

$$a(t) = 1 + st = 1 + 0.1t \implies a(3) = 1 + 0.1(3)$$

For the next five years, compound interest of 8%

$$a(t) = (1+i)^t = (1.08)^t \implies a(5) = (1.08)^5$$

For the last two years, compound discount of 6%

$$a(t) = (1-d)^{-t} = (1-0.06)^{-t} \implies a(2) = (0.94)^{-2}$$

$$Answer = (10,000)(1+0.1(3))(1.08)^5(0.94)^{-2} = 21,617.55$$

You are given that $v(t) = \dfrac{1}{\alpha + \beta t^2}$.

Under this discount function, 500 at time 10 has a present value of 250.

Determine $a(20)$.

Solution:

$$v(t) = \frac{1}{a(t)} \text{==>} a(t) = \alpha + \beta t^2$$

$$a(0) = 1 \text{==>} \alpha + \beta(0)^2 = 1 \text{==>} \alpha = 1$$

$$250a(10) = 500 \text{==>} a(10) = 2 \text{==>} 1 + \beta(10)^2 = 2 \text{==>} \beta(100) = 1 \text{==>} \beta = 0.01$$

$$a(t) = 1 + 0.01t^2 \text{==>} a(20) = 1 + 0.01(20)^2 = 5$$

Let i_{10} be the effective interest rate in the 10th year for simple interest at a simple interest rate of 7%.

Let d_{10} be the effective discount rate in the 10th year under compound interest at an annual effective interest rate of 4%.

Calculate $i_{10} - d_{10}$. (Provide your answer to five decimal places.)

Solution:

$$i_{10} = \frac{a(10) - a(9)}{a(9)} = \frac{1 + 0.07(10) - \left[1 + 0.07(9)\right]}{1 + 0.07(9)} = \frac{0.07}{1.63} = 0.042944785$$

or under simple interest

$$i_n = \frac{s}{1 + (n-1)s} \implies i_{10} = \frac{0.07}{1 + (10-1)(0.07)} = \frac{0.07}{1.63} = 0.042944785$$

$$d_{10} = \frac{a(10) - a(9)}{a(10)} = \frac{(1.04)^{10} - \left[(1.04)^{9}\right]}{(1.04)^{10}} = 0.038461538$$

or under compound interest d is constant so $d_{10} = d$

$$d = \frac{i}{1+i} = \frac{0.04}{1.04} = 0.038461538$$

$$i_{10} - d_{10} = 0.042944785 - 0.038461538 = 0.00448$$