Topic: Dividend Discount Method

The preferred stock of Wiser Corporation pays quarterly dividends of 1.25 with the next dividend to be paid in 1 month.

Using an interest rate of 12% compounded quarterly and the dividend discount method, determine the price of the preferred stock.

Solution:

We need $\frac{i^{(4)}}{4}$. We are given $i^{(4)} = 0.12$, so $\frac{i^{(4)}}{4} = \frac{0.12}{4} = 0.03$.

$$Price = PV = \left(\frac{1.25}{0.03}\right)(1.03)^{2/3} = 42.496$$

We multiply by $(1.03)^{2/3}$ because the first dividend is paid at the end of 1 month.

The common stock of Beckley Corporation pays quarterly dividends. The next dividend of 2 will be paid in 3 months. Future dividends are expected to increase with a dividend of 2(1.01) being paid in 6 months. The dividend to be paid in 9 months will be $2(1.01)^2$. Dividends will continue to increase in the same pattern.

Using the dividend discount method with an annual effective interest rate of 12%, determine the price of the common stock of Beckley Corporation.

Solution:

We need $\frac{i^{(4)}}{4}$. We are given $i = 0.12$, so $\frac{i^{(4)}}{4} = (1.12)^{\frac{1}{4}} - 1 = 0.028737345$.

$$Price = PV = 2(1.028737345)^{-1} + 2(1.01)(1.028737345)^{-2} + 2(1.01)^2(1.028737345)^{-3} + \cdots$$

$$= \frac{2(1.028737345)^{-1} - 0}{1 - (1.01)(1.028737345)^{-1}} = 106.74$$

The common stock of Mills Company pays a quarterly dividend. The next dividend of 3 is payable in 3 months. Subsequent dividends are expected to each be 0.15 greater than the prior dividend. In other words, the second dividend will be 3.15, the third dividend will be 3.30, etc.

Using the dividend discount method with an annual effective interest rate of i, the stock price is 360.

Determine i.

Solution:

$$Price = PV = 360 = \frac{3.00}{\frac{i^{(4)}}{4}} + \frac{0.15}{\left(\frac{i^{(4)}}{4}\right)^2} \quad ==> \quad 360\left(\frac{i^{(4)}}{4}\right)^2 - 3.00\frac{i^{(4)}}{4} - 0.15 = 0$$

Solve for $\frac{i^{(4)}}{4}$:

$$\frac{i^{(4)}}{4} = \frac{3.00 + \sqrt{(-3.00)^2 - 4(-0.15)(360)}}{2(360)} = 0.025$$

Solve for i:

$$i = \left(1 + \frac{i^{(4)}}{4}\right)^4 - 1 = (1.025)^4 - 1 = 0.103812891$$

Jackson buys the common stock of The Jones Company. The Jones Company is rapidly growing and its dividends are expected to increase into the future. The dividends are payable quarterly. The first dividend payable in two months and is expected to be 1.00. The next dividend is payable in 5 months and is expected to be 1.10. The dividends will continue to increase with each dividend being 0.10 greater than the previous dividend.

Jackson wants to earn a yield of 10% compounded quarterly.

Using the dividend discount method, determine the price that Jackson should pay for the stock of The Jones Company.

Solution:

Dividends are quarterly so we need $\frac{i^{(4)}}{4} = \frac{0.10}{4} = 0.025.$

Payments increase arithmetically so we need P&Q.

$$PV = \left(\frac{1.00}{0.025} + \frac{0.10}{(0.025)^2} \right)(1.025)^{1/3} = 201.65$$

The $(1.025)^{1/3}$ is because the next payment is in 2 months.

Mattheos buys the preferred stock of Mercer Corporation. The preferred stock pays quarterly dividends. The next dividend paid later today will be 8.00. All future dividends are expected to be 8.00.

Mattheos expects an annual effective yield on the preferred stock of 12%.

Using the dividend discount method, calculate the price that Mattheos should pay for the preferred stock.

Solution:

The dividends are level with the next dividend paid later today. Therefore, we use the formula for a perpetuity due. Dividends are quarterly to we need $\frac{i^{(4)}}{4}$. We are given i.

$$(1+i)=\left(1+\frac{i^{(4)}}{4}\right)^4 \Longrightarrow \frac{i^{(4)}}{4}=(1.12)^{0.25}-1=0.028737345$$

$$PV=\left(\frac{8}{0.028737345}\right)(1.028737345)=286.38$$

The stock of Irwin Investment Company sells for 256.25. The stock pays quarterly dividends with the next dividend due in 3 months. The dividend payable in three months is expected to be 4.00. Dividends thereafter are expected to increase with the second dividend at the end of 6 months being 4.25 and the dividend at the end of 9 months being 4.50. The dividends will continue to increase with each dividend being 0.25 greater than the previous dividend.

The stock of Irwin is expected to yield an annual effective yield rate of i based on the dividend discount method.

Determine i.

Solution:

$$Price = PV = 256.25 = \frac{4}{\frac{i^{(4)}}{4}} + \frac{0.25}{\left(\frac{i^{(4)}}{4}\right)^2} \Rightarrow 256.25\left(\frac{i^{(4)}}{4}\right)^2 - 4\left(\frac{i^{(4)}}{4}\right) - 0.25 = 0$$

$$\frac{i^{(4)}}{4} = \frac{-b \pm \sqrt{(b)^2 - (4)(a)(c)}}{(2)(a)} = \frac{4 \pm \sqrt{(-4)^2 - 4(256.25)(-0.25)}}{2(256.25)} = 0.04$$

$$i = \left(1 + \frac{i^{(4)}}{4}\right)^4 - 1 = (1.04)^4 - 1 = 0.16985856$$

Guilia decides to buy two stocks. Both stocks are bought to yield 20% compounded quarterly.

The first stock is the preferred stock of Gao Corporation. This stock pays a dividend of 10 each quarter with the next dividend being payable in two months.

The second stock is the common stock of Thompson Corporation. The next dividend for Thompson Corporation is paid at the end of three months and will be 8.50. Thompson is a rapidly growing company and dividends are expected to increase in the future with each dividend being 103% of the prior dividend. In other words, the first dividend will be 8.50, the second dividend will be 8.50(1.03), the third dividend will be $8.50(1.03)^2$, etc.

Using the dividend discount method, calculate the total amount that Guilia will pay to purchase these two stocks.

Solution:

Since payments are made quarterly, we need $\frac{i^{(4)}}{4}$ and we have $i^{(4)}$.

$$\frac{i^{(4)}}{4} = \frac{0.20}{4} = 0.05$$

$$Stock\ 1: Price = PV = \left(\frac{10}{0.05}\right)(1.05)^{1/3} = 203.28$$

We multiply by $(1.05)^{1/3}$ because the next dividend is due in 2 months.

$$\begin{aligned} Stock\ 2: Price &= PV \\ &= 8.50(1.05)^{-1} + 8.05(1.03)(1.05)^{-2} + 8.50(1.03)^2(1.05)^{-3} + \cdots \\ &= \left(\frac{8.50(1.05)^{-1} - 0}{1 - (1.03)(1.05)^{-1}}\right) = 425.00 \end{aligned}$$

$$Total\ Price = 203.28 + 425.00 = 628.28$$

The stock of Bray Industries pays a quarterly dividend with the next dividend payable in 2 months. The first dividend will be 10. The second dividend will be 11. The third dividend will be 12. Each dividend will follow the same pattern with each dividend being 1 greater than the prior dividend.

Using the dividend discount method, determine the price to yield 10% compounded quarterly.

Solution:

We need $\frac{i^{(4)}}{4}$. We are given that $i^{(4)} = 0.10$ so $\frac{i^{(4)}}{4} = 0.025$.

$$PV = \left(\frac{10}{0.025} + \frac{1}{0.025} \right)(1.025)^{1/3} = 2016.53$$

We multiply by $(1.025)^{1/3}$ because the first divdend is paid at the end of two months.

The stock of Crouthamel Company pays quarterly dividends with the next dividend of 4 being paid later today. Each dividend thereafter increases 1.5% of the prior dividend. In other words, the second dividend at the end of three months will be $4(1.015)$. The third dividend paid at the end of six months will be $4(1.015)^2$, etc.

Using the dividend discount method, determine the price of Crouthamel stock at an annual effective discount rate of 12%.

Solution:

We need $\frac{i^{(4)}}{4}$. We are given that $i = 0.12$ so $\frac{i^{(4)}}{4} = (1.12)^{0.25} - 1 = 0.028737345$

$$PV = 4 + 4(1.015)(1.028737345)^{-1} + 4(1.015)^2(1.028737345)^{-2} + ...$$

$$= \frac{4-0}{1-(1.015)(1.028737345)^{-1}} = 299.55$$

Because of poor wording, some students interpreted the question to say that $d = 0.12$. Then

$$i = \frac{d}{1-d} = \frac{0.12}{1-0.12} = 0.136363636 ==> \frac{i^{(4)}}{4} = (1.136363636)^{0.25} - 1 = 0.032474494$$

$$PV = 4 + 4(1.015)(1.032474494)^{-1} + 4(1.015)^2(1.032474494)^{-2} + ...$$

$$= \frac{4-0}{1-(1.015)(1.032474494)^{-1}} = 236.34$$

The stock of Widmer Corporation pays a quarterly dividend with the next dividend to be paid in two months. The next dividend paid in two months is expected to be 5. The second dividend paid in five months is expected to be 5(1.02). The third dividend paid in 8 months is expected to be $5(1.02)^2$. Dividends are expect to continue in the same pattern with each dividend being 102% of the previous dividend.

Simon wants to buy the stock to yield 15% annually.

Using the dividend discount method, determine the price of this stock.

Solution:

Since payments are made quarterly, we need $\frac{i^{(4)}}{4}$ and we have $i = 0.15$.

$$\frac{i^{(4)}}{4} = (1.15)^{0.25} - 1 = 0.035558$$

$$\text{Price} = PV = \left[5(1.035558)^{-1} + 5(1.02)(1.035558)^{-2} + 5(1.02)^2(1.035558)^{-3} + ... \right](1.035558)^{1/3}$$

$$= \left[\frac{5(1.035558)^{-1} - 0}{1 - (1.02)(1.035558)^{-1}} \right](1.035558)^{1/3} = 325.14$$

Lauren can purchase Ginuli Stock for 200 or Alex Stock for 200. Both stocks are expected to provide the same annual yield. Both prices are determined using the dividend discount method.

Ginuli Stock pays a level quarterly dividend of 5 with the next dividend paid later today.

Alex Stock is expected to pay an increasing dividend with the next dividend equal to D paid in three months. The second dividend paid in six months is expected to be $D+0.05$. The third dividend paid in nine months is expected to be $D+0.10$. Each dividend thereafter is expected to have the same pattern with the dividend being 0.05 greater than the previous dividend.

Determine D.

Solution:

$$\text{Let } i = \frac{i^{(4)}}{4}$$

$$Ginuili ==> \left(\frac{5}{i}\right)(1+i) = 200 ==> \frac{5}{i} + 5 = 200 ==> i = \frac{5}{195}$$

$$Alex ==> \frac{D}{i} + \frac{0.05}{i^2} = 200 ==> \frac{D}{\left(\frac{5}{195}\right)} + \frac{0.05}{\left(\frac{5}{195}\right)^2} = 200 ==> D = \left[200 - \frac{0.05}{\left(\frac{5}{195}\right)^2}\right]\left(\frac{5}{195}\right) = 3.18$$

The preferred stock of Patterson Industries pays a quarterly dividend of 2.45 with the next dividend being paid in 2 months.

Using the dividend discount method, determine the price of this preferred stock to yield 8% compounded quarterly.

Solution:

Dividends are quarterly so we need $\frac{i^{(4)}}{4}$. We are given $i^{(4)} = 0.08$ so $\frac{i^{(4)}}{4} = \frac{0.08}{4} = 0.02$.

$$\text{Price} = PV = 2.45\frac{1}{0.02}(1.02)^{1/3} = 123.31$$

We multiply by $(1.02)^{1/3}$ because the first dividend is payable in two months.

The common stock of Hibbler LTD pays a quarterly dividend with the next dividend of 4.00 to be paid in three months. Hibbler is a rapidly growing company and dividends are expected to increase each quarter. The amount of the increase is expected to be 2% per quarter. In other words, the first dividend will be 4.00. The second dividend will be $4.00(1.02)$. The third dividend will be $4.00(1.02)^2$. The dividends will continue to increase in this pattern.

Using the dividend discount method, determine the price of this common stock to yield a 12% annual effective rate.

Solution:

Dividends are quarterly so we need $\frac{i^{(4)}}{4}$. We are given $i = 0.12$

so $\frac{i^{(4)}}{4} = (1.12)^{1/4} - 1 = 0.028737345.$

$$\text{Price} = PV = 4(1.028737345)^{-1} + 4(1.02)(1.028737345)^{-2} + \ldots$$

$$\frac{4(1.028737345)^{-1} - 0}{1-(1.02)(1.028737345)^{-1}} = 457.80$$

The common stock of LaMaster Corporation pays a quarterly dividend with the next dividend of 30.00 being paid later today. Each dividend is expected to be 0.25 larger than the prior dividend. In other words, the first dividend will be 30.00, the second divided will be 30.25, the third dividend will be 30.50, etc.

Using the dividend discount method, determine the price of this common stock to yield a 12% compounded quarterly.

Solution:

Dividends are quarterly so we need $\frac{i^{(4)}}{4}$. We are given $i^{(4)} = 0.12$ so $\frac{i^{(4)}}{4} = \frac{0.12}{4} = 0.03$.

We also note that this is a perpetuity due since the next dividend is paid later today.

$$\text{Price} = PV = \left[\frac{30}{0.03} + \frac{0.25}{(0.03)^3} \right](1.03) = 1316.11$$

We multiply by (1.03) because the first dividend is payable today.

Saqqa Corporation pays quarterly dividends with the next dividend of 4.00 being paid in one month. Saqqa is a rapidly growing company and expects each dividend to increase by 3% over the previous dividend. In other words, the dividend paid in one month will be 4.00, the dividend paid in four months will be $4.00(1.03)$, the dividend to be paid in seven months will be $4.00(1.03)^2$, etc.

Using the dividend discount method, determine the price of a share of Saqqa stock using an interest rate of 20% compounded quarterly.

Solution:

We need $\frac{i^{(4)}}{4}$ which is $\frac{0.20}{4} = 0.05.$

$$PV = (4.00v + 4.00(1.03)v^2 + ...)(1.05)^{2/3}$$

$$= \frac{4(1.03)^{-1} - 0}{1-(1.03)(1.05)^{-1}}(1.05)^{2/3} = 206.61$$

The stock of White Industries pays quarterly dividends. The next dividend is payable in three months and will be 5.00. The dividends for White Industries are expected to increase with each dividend being 0.10 larger than the previous dividend. In other words, the first dividend will be 5.00. The second dividend will be 5.10. The third dividend will be 5.20, etc.

Alisa purchases the stock for 200.

Using the dividend discount method, determine the annual effective interest rate that Alisa expects to earn on the stock.

Solutions:

$$PV = 200 = \frac{5}{i} + \frac{0.1}{i^2} ==> 200i^2 - 5i - 0.1 = 0$$

$$i = \frac{5 \pm \sqrt{(-5)^2} - 4(200)(-0.10)}{2(200)} = 0.038117377$$

$$\text{Since payments are quarterly, } \frac{i^{(4)}}{4} = 0.038117377$$

$$i = (1.038117377)^4 - 1 = 0.1614108$$

The common stock of Zhang Corporation pays a quarterly dividend. The next dividend of 5.00 will be paid in one month. Future dividends are expected to increase such that each dividend is 2% greater than the prior dividend. In other words, a dividend of 5.00 will be paid at the end of one month. A dividend of $5.00(1.02)$ will be paid at the end of 4 months. A dividend of $5.00(1.02)^2$ will be paid at the end of 7 months, etc.

Using the dividend discount method, determine the price that Summer should pay in order to have an annual effective return of 12%.

Solution:

$$PV = 5(1.12)^{-1/12} + 5(1.02)(1.12)^{-4/12} + \ldots$$

$$= \frac{5(1.12)^{-1/12}}{1-(1.02)(1.12)^{-3/12}} = 583.17$$

Chufan is considering the following two investments:

a. A preferred stock issued by Osborn LTD. The preferred stock pays quarterly dividends of 25 with the next dividend payable later today. The price of the stock is 1000.

b. A bond issued by Johnson Inc. The bond has a par value of 1000. The coupons are paid semi-annually at a rate of 8.6% compounded semi-annually. The bond matures for 1500 at the end of 20 years.

The preferred stock of Osborn LTD and the bond of Johnsons Inc are expected to provide the same **annual effective** interest rate.

Determine the price of the bond.

Solution:

From the stock,

$$\text{Price}=1000=\frac{25}{\frac{i^{(4)}}{4}}\left(1+\frac{i^{(4)}}{4}\right)=\frac{25}{\frac{i^{(4)}}{4}}+25 \Longrightarrow \frac{i^{(4)}}{4}=\frac{25}{1000-25}=0.025641$$

$$i=\left(1+\frac{i^{(4)}}{4}\right)^4-1=0.106577$$

But we need $\frac{i^{(2)}}{2}$ for the bond so $\frac{i^{(2)}}{2}=(1+i)^{0.5}-1=(1.106577)^{0.5}-1=0.05193964$

$$P=(1000)(0.086/2)\left(\frac{1-(1.05193964)^{-40}}{0.051939964}\right)+1500(1.05193964)^{-40}=916.56$$

or

$$\boxed{PMT}\leftarrow(1000)(0.043)=43;\boxed{FV}\leftarrow 1500;\boxed{I/Y}\leftarrow 5.193964;\boxed{N}\leftarrow 40$$

$$\boxed{CPT}\boxed{PV}\rightarrow 916.56$$