## Topic: Drop and Balloon Payments

Liu Manufacturing borrows 1,000,000 to be repaid with annual payments of 90,000 followed by a drop payment. The annual effective interest rate on the loan is $6.3 \%$.

Determine the amount of the drop payment.

## Solution:

Using the BA-II+ Calculator:

$$
\begin{aligned}
& X V \leftarrow 1,000,000 ; \quad I / Y \leftarrow 6.3 ; P M T \leftarrow-90,000 \\
& C P T N \Rightarrow 19.7065
\end{aligned}
$$

Round down to 19

$$
\text { Drop }=(60,353.61)(1.063)=64,155.89
$$

Or mathematically:
$1,000,000(1.063)^{20}-90,000 \ddot{s}_{19}=64,155.89$
*Logic check: your solution should be less than the amount of a single payment $(90,000)$

Mattie is the beneficiary of a perpetuity due with quarterly payments of 1000. The present value of Mattie's perpetuity is 40,000.

Mitch borrows 500,000 which will be repaid with quarterly payments of 16,000 plus a drop payment. The interest rate on the loan is the same as the interest rate on Mattie's perpetuity.

Calculate the drop payment on Mitch's loan.

## Solution:

## Mattie

Note that it is a perpetuity due with quarterly payments
$1000\left(\frac{1}{\frac{i^{(4)}}{4}}\right)\left(1+\frac{i^{(4)}}{4}\right)=40,000==>\left(\frac{1000}{\frac{i^{(4)}}{4}}\right)+1000=40,000==>\frac{i^{(4)}}{4}=\frac{1}{39}=0.025641026$

## Mitch

We note that Mitch's payments are also quarterly so we just use $\frac{i^{(4)}}{4}$.
$I / Y \leftarrow 2.5641026<==$ Note that this must go in as a percent and not as a decimal.
$P V \leftarrow 500,000$
$P M T \leftarrow-16,000$

| $C P T$ | $N$ |
| :---: | :---: |
| 63.823 | Round down to 63 |


| $2 n d$ | Amort $P 1$ | $P 2$ |
| :---: | :---: | :---: |

Drop $=(12,873.81)(1.025641026)=13,203.91$
Or mathematically
$500,000(1.025641026)^{64}-16,000 \ddot{3}_{63}=13,203.91$
*Logic check: your solution should be less than the amount of a single payment $(16,000)$

Jacque borrows 100,000 and is repaying it with annual payments of 9000 plus a balloon payment. The annual effective interest rate on the loan is $5 \%$.

Calculate the balloon payment.

## Solution:

$$
\begin{aligned}
& P V=100,000 \\
& P M T=-9000 \\
& I / Y=5 \\
& C P T \text { N }=16.62 \\
& 2 \text { nd Amort } \\
& P 1=1 \\
& P 2=16 \\
& \text { Bal }=5370.03 \\
& \text { Ballon }=9000+5370.03=14,370.03
\end{aligned}
$$

Or mathematically
$100,000(1.05)^{16}-9000 \ddot{s}_{15}=14,370.03$
*Logic check: your solution should be greater than the amount of a single payment ( 9,000 ), but less than double the amount of a single payment $(18,000)$

Giacomo Inc. borrows 60,000 which will be repaid with monthly payments of 500 plus a balloon payment of $B$. The interest rate on the loan is $5.8 \%$ compounded monthly.

Freija has an identical loan except her loan calls for the final payment to be a drop payment of $D$.

Determine $B-D$.

## Solution:

Use your calculator to find $n$.

$$
P V \leftarrow 60,000 ; I / Y \leftarrow 5.8 / 12=0.48333333 ; P M T \leftarrow-500
$$

$$
\begin{array}{|c|}
\hline C P T
\end{array} \rightarrow 179.916==>\text { round down to } 179
$$

$$
\begin{array}{|l|l|ll}
\hline 2 n d & \text { Amort } & P 1 & \leftarrow 1 ; P 2 \leftarrow 179 ; ~ \\
\mathrm{Bal}
\end{array} 456.0271744
$$

$$
\text { Balloon }=\text { Bal }+Q=456.03+500=956.03
$$

$$
\text { Drop }=(\text { Bal })(1.00483333333)=458.23
$$

$$
B-D=956.03-458.23=497.80
$$

Or mathematically

$$
\begin{aligned}
& B=60,000(1.04833333)^{179}-500 \ddot{s}_{178}=956.03 \\
& D=60,000(1.04833333)^{180}-500 \ddot{s}_{\overline{179}}=458.23 \\
& B-D=956.03-458.23=497.80
\end{aligned}
$$

*Logic check: your solution for $B$ should be greater than the amount of a single payment (500), but less than double the amount of a single payment $(1,000)$; your solution for $D$ should be less than the amount for a single payment (500)

