Topic: Duration and Convexity of a Portfolio

You have the following portfolio of bonds:

a. A zero coupon bond which matures for 60,000 at the end of 9 years.

b. A 12 year bond with a Modified Convexity of 80 and a price of 40,000.

Calculate the Modified Convexity for this portfolio of bonds at an interest rate of 5%.

Solution:

$\underline{Bond\ A:}$

$Since\ this\ is\ a\ zero\ coupon\ bond.$

$MacDur = 9\ , \quad MacCon = 9^2 = 81$

$$ModCon = (MacDur + MadCon)v^2 = \frac{(9+81)}{(1.05)^2} = 81.63265306$$

$$Price = \frac{60{,}000}{(1.05)^9} = 38{,}676.53$$

$\underline{Bond\ B:}$

$ModCon = 80, \quad Price = 40{,}000$

$$C_{ModCon}^{Port} = \frac{\Sigma P^t Ct}{\Sigma P^t} = \frac{(38{,}676.53)(81.63265306) + (40{,}000)(80)}{38{,}676.53 + 40{,}000}$$

$$= 80.8026$$

The Bray Insurance Company has the following portfolio of annuities that it will be paying over the next several years:

	Present Value of Future Payments	**Modified Duration**
Annuity 1	400,000	8
Annuity 2	350,000	10
Annuity 3	250,000	14

Christine who is the actuary for Bray, wants to spend 1,000,000 to purchase bonds. Christine can purchase the following two bonds in any amount. (In other words, she can purchase partial bonds.)

	Modified Duration
Bond 1	6
Bond 2	18

Christine's objective in buying bonds is to match the Modified Duration of the annuity portfolio.

Determine the amount that Christine should spend on Bond 1.

Solution:

For the annuities:

$$D_{ModDur}^{Port} = \frac{(400,000)(8) + (350,000)(10) + (250,000)(14)}{400,000 + 350,000 + 250,000} = 10.2$$

For the Assets:

$$D_{ModDur}^{Port} = \frac{(X)(6) + (1,000,000 - X)(18)}{1,000,000} = 10.2$$

$$10,200,000 = 18,000,000 - 12X$$

$$X = \frac{18,000,000 - 10,200,000}{12} = 650,000$$

Kevin owns the following bonds. All values are calculated using an annual effective interest rate of 5%.

	Macaulay Duration	Macaulay Convexity	Price
Bond A	10	90	30,000
Bond B	16	200	45,000
Bond C	8	55	25,000

Determine the modified convexity of Kevin's bond portfolio at an annual effective interest rate of 5%

Solution:

$$ModCon \text{ for Bond A} = (10+90)v^2 = 100v^2$$

$$ModCon \text{ for Bond B} = (16+200)v^2 = 216v^2$$

$$ModCon \text{ for Bond C} = (8+55)v^2 = 63v^2$$

$$C_{ModCon}^{Port} = \frac{\sum P^t \cdot C^t}{\sum P^t} = \frac{(30,000)(100)(1.05)^{-2} + (45,000)(216)(1.05)^{-2} + (25,000)(63)(1.05)^{-2}}{30,000+45,000+25,000}$$

$$= 129.66$$

The Moses Life Insurance Company owns the following two bonds:

	Price	Macaulay Duration	Macaulay Convexity
Bond A	60,000	A	90
Bond B	40,000	12	125

Using an interest rate of 8%, the Modified Convexity of this bond portfolio is 98.654.

Determine A.

Solution:

Bond A

$$ModCon = \frac{MacDur + MacCon}{(1.08)^2} = \frac{A+90}{(1.08)^2}$$

Bond B

$$ModCon = \frac{MacDur + MacCon}{(1.08)^2} = \frac{12+125}{(1.08)^2} = 117.4554$$

$$ModConPort = \frac{P^A \cdot C^A + P^B \cdot C^B}{P^A + P^B} = \frac{(60,000)\left(\frac{A+90}{(1.08)^2}\right) + (40,000)(117.4554)}{60,000 + 40,000} = 98.654$$

$$==> \frac{A+90}{(1.08)^2} = \frac{98.654(100,000) - (40,000)(117.4554)}{60,000} = 86.1197$$

$$A = 86.1197(1.08)^2 - 90 = 10.45$$