Topic: Equations of Value

Seth borrows 1000 to be repaid at an annual interest rate of 7.2%. Seth will make payments of 2P at time 3 and P at time 5.

Determine P.

Solution:

$$1000 = 2P(1.072)^{-3} + P(1.072)^{-5}$$

$$P = \frac{1000}{2(1.072)^{-3} + (1.072)^{-5}} = 429.21$$

OR

$$1000(1.072)^5 = 2P(1.072)^2 + P$$

$$P = \frac{1000(1.072)^5}{2(1.072)^2 + 1} = 429.21$$

Kiersten borrows 20,000 from Sarah at a nominal annual interest rate of 9% compounded monthly.

Kiersten will repay the loan with two payments of P. The first payment of P will be at the end of three years and the second payment of P will be at the end of seven years.

Determine P.

Solution:

$$20,000\left(1+\frac{0.09}{12}\right)^{(12)(7)} = P\left(1+\frac{0.09}{12}\right)^{(12)(4)} + P$$

$$P = \frac{20,000\left(1+\frac{0.09}{12}\right)^{(12)(7)}}{\left(1+\frac{0.09}{12}\right)^{(12)(4)}+1} = 15,408.39$$

Walker borrows 25,000 at an annual effective interest rate of 8%. Walker will repay the loan with three payments over the next five years. The first payment will be $3P$ at the end of 2 years. The second payment will be P at the end of 3 years. The final payment will be $2P$ at the end of five years.

Determine P.

Solution:

$$25,000 = 3P(1.08)^{-2} + P(1.08)^{-3} + 2P(1.08)^{-5}$$

$$P = \frac{25,000}{3(1.08)^{-2} + (1.08)^{-3} + 2(1.08)^{-5}} = 5288.75$$

Or

$$25,000(1.08)^5 = 3P(1.08)^3 + P(1.08)^2 + 2P$$

$$P = \frac{25,000(1.08)^5}{3(1.08)^3 + (1.08)^2 + 2} = 5288.75$$

Bri borrows 5000 which will be repaid with a payment of P at the end of one year and $2P$ at the end of three years. The annual effective interest rate on this loan is 6%

Determine P .

Solutions:

$$5000 = P(1.06)^{-1} + 2P(1.06)^{-3}$$

$$P = \frac{5000}{(1.06)^{-1} + 2(1.06)^{-3}} = 1906.48$$

OR

$$5000(1.06)^3 = P(1.06)^2 + 2P$$

$$P = \frac{5000(1.06)^3}{(1.06)^2 + 2} = 1906.48$$

Billy borrows 10,000 to be repaid with three payments. The first payment is P at the end of two years. The second payment is $2P$ at the end of 4 years. The final payment is $3P$ at the end of five years.

The loan has an annual effective interest rate of 6%.

Determine P.

Solution:

$$(10,000)(1.06)^5 = P(1.06)^3 + 2P(1.06) + 3P$$

$$P = \frac{(10,000)(1.06)^5}{(1.06)^3 + 2(1.06) + 3} = 2120.46$$