Topic: First Order Price Approximation

Tong LTD owns two bonds. The bonds have the following characteristics:

	Price	Modified Convexity	Macaulay Convexity
Bond A	40,000	58	60
Bond B	60,000	86	90

The above values are all based on an annual effective interest rate of 8%.

Using the First Order Macaulay Approximation, calculate the estimated price of this bond portfolio if the annual effective interest rate is 7%.

Solution:

$$P(i) = P(i_0)\left[\frac{1+i_0}{1+i}\right]^{MacDur}$$

$$P(i_0) = 40,000 + 60,000 = 100,000$$

$$i = 0.07 \text{ and } i_0 = 0.08$$

$$ModCon = v^2(MacDur + MacCon) ==> 58 = (1.08)^{-2}(MacDur + 60)$$

$$==> MacDur \text{ Bond A=7.6512}$$

$$ModCon = v^2(MacDur + MacCon) ==> 86 = (1.08)^{-2}(MacDur + 90)$$

$$==> MacDur \text{ Bond B=10.3104}$$

$$D_{MacDur}^{Port} = \frac{(40,000)(7.6512) + (60,000)(10.3104)}{100,000} = 9.2467$$

$$P(i) = (100,000)\left[\frac{1.08}{1.07}\right]^{9.2467} = 108,982.42$$

A 10 year bond has a price of 24,000. The Macaulay duration of the bond is 12. The Modified convexity of the bond is 123. All values are calculated using an annual effective interest rate of 4.5%.

$P_1 =$ the estimated price of this bond using the first order Macaulay approximation at an annual effective interest rate of 5.75%.

$P_2 =$ the estimated price of this bond using the first order Modified approximation at an annual effective interest rate of 5.75%.

Determine $P_1 - P_2$.

Solution:

$$ModDur = MacDur(1.045)^{-1} = (12)(1.045)^{-1} = 11.4832536$$

$$P_1 = (24,000)\left[\frac{1.045}{1.0575}\right]^{12} = 20,808.57$$

$$P_2 = (24,000)(1-(0.0575-0.045)(11.4832536)) = 20,555.02$$

$$P_1 - P_2 = 20,808.57 - 20,555.02 = 253.55$$

Graham owns a bond with a price of 130,000 at an annual effective yield rate of 7%. The bond has a Modified Duration of 14.0187 and a Modified Convexity of 100 at an annual effective interest rate of 7%.

Graham estimates that price of this bond at an interest rate of i using the first order Macaulay approximation to be 117,886.61.

Determine i.

Solution:

First Order Macaulay Approximation

$$P(i) = P(i_0)\left[\frac{1+i_0}{1+i}\right]^{MacDur}$$

$$ModDur = V(MacDur) \Longrightarrow 14.0187 = (1.07)^{-1}(MacDur) \Longrightarrow MacDur = (14.0187)(1.07) = 15$$

$$P(i) = 117,886.61 = (130,000)\left[\frac{1.07}{1+i}\right]^{15} \Longrightarrow \frac{1.07}{1+i} = \left[\frac{117,886.61}{130,000}\right]^{\frac{1}{15}} = 0.993500466$$

$$\Longrightarrow 1+i = \frac{1.07}{0.993500466} = 1.077 \Longrightarrow i = 0.077$$