Topic: Immunization

The Filza Insurance Company owes Mary 10,000 at the end of one year and 30,000 at the end of two years. Filza uses exact matching to protect from interest rate changes.

Filza uses the following two bonds to exactly match the cash flows:

a. Bond A is a one year bond with annual coupons of 100 and a maturity value of 1600.

b. Bond B is a two year bond with annual coupons of 60 and a maturity value of 1000.

Determine the number of each bond that Filza should purchase.

Solution:

	Time 1	**Time 2**
Payments to Mary	10,000	30,000
Bond A	1700	
Bond B	60	1060

$$30,000 = 1060B \Longrightarrow B = \frac{30,000}{1060} = 28.30189$$

$$10,000 = 60B + 1700A = 60(28.30189) + 1700A$$

$$\Longrightarrow A = \frac{10,000 - (60)(28.30189)}{1700} = 4.88346$$

Kimberly has agreed to pay Aashima a payment of 25,000 at the end of one year and an additional payment of 42,000 at the end of two years.

In order to protect herself from interest rate changes, Kimberly wants to absolutely match the cash flows using the following two bonds:

a. Bond 1 is a one year bond with annual coupons of 50 and a maturity value of 1200. The price of this bond is 1150.

b. Bond 2 is a two year bond with annual coupons of 100 and a maturity value of 2000. The annual effective yield rate on this bond is 5%.

Determine the number of each of the bonds that Kimberly will need to purchase. Assume that Kimberly can buy partial bonds.

Solution:

$$Time\ 1 \Rightarrow (Bond1)(1{,}250) + (Bond2)(100) = 25{,}000$$

$$Time\ 2 \Rightarrow (Bond1)(0) + (Bond2)(2{,}100) = 42{,}000$$

$$Using\ Time\ 2 \Rightarrow Bond2 = \frac{42{,}000}{2{,}100} = 20$$

$$Using\ Time\ 1 \Rightarrow Bond1 = \frac{25{,}000 - (20)(100)}{1{,}250} = 18.4$$

The Thea Insurance Company has agreed to pay 500,000 to Karinna at the end of six years. The annual effective interest rate is 4%. In order to protect against interest rate changes, Thea has decided to fully immunize the payment using the following to zero coupon bonds:

a. Bond A matures for 10,000 at the end of 3 years.

b. Bond B matures for 25,000 at the end of 7 years.

Determine the number of Bond B that Thea should purchase. Assume that Thea can buy partial bonds.

Solution:

$$AmountBondB = \frac{D^L - D^A}{D^B - D^A}(PV\ of\ Liability) = \left(\frac{6-3}{7-3}\right)(500{,}000)(1.04)^{-6}$$

$$= 296{,}367.9471$$

$$NumberBondB = \frac{AmountBondB}{PriceBondB} = \frac{296{,}367.9471}{(25{,}000)(1.04)^{-7}} = 15.6$$

Liu Life Insurance Company has promised to pay Fang a payment of 500,000 at the end of 8 years.

Liu Life Insurance Company wants to immunize the payment to Fang by investing the following two assets:

i. Asset A is a zero coupon bond maturing for 10,000 at the end of 2 years.
ii. Asset B is a zero coupon bond maturing for 10,000 at the end of 10 years.

At an annual effective interest rate of 7%, calculate the number of each bonds that Liu should buy. (Assume that you can buy partial bonds.)

Solution:

Total price of asset A = $P_A = \left(\frac{500,000}{(1+i)^8}\right)\left(\frac{10-8}{10-2}\right) = 72,751.13807$

price per A = $\frac{10,000}{(1.07)^2} = 8734.387283$

Number of bond A = $\frac{72,751.13807}{8734.387283} = 8.329277797$

Total price of Asset B = $P_B = \left(\frac{500,000}{(1.07)^8}\right)\left(\frac{6}{8}\right) = 218,253.4142$

Price of Asset B = $\frac{10,000}{(1.07)^{10}} = 5083.492921$

Number of Asset B = $\frac{218,253.4142}{5083.492921} = 42.93375$

Tomas has agreed to pay the following payments to Taylen:

a. 100,000 at the end of one year;

b. 250,000 at the end of two years; and

c. 400,000 at the end of four years.

Tomas wants to exactly match the payments using the following bonds:

i. Bond 1 is a one year bond with annual coupons of 100 and a maturity value of 1200.

ii. Bond 2 is a two year bond with annual coupons of 80 and a maturity value of 1000.

iii. Bond 3 is a three year bond with annual coupons of 400 and a maturity value of 10,000

iv. Bond 4 is a zero coupon bond maturing for 100,000 at the end of four years.

Determine the number of Bond 1 that Tomas should buy. Assume that you can buy partial bonds.

Solution:

	Time 1	Time 2	Time 3	Time 4
Payments to Mary	100,000	250,000	0	400,000
Bond 1	1300			
Bond 2	80	1080		
Bond 3	400	400	14,000	
Bond 4	0	0	0	100,000

$$Bond4 ==> 100,000(Bond4) = 400,000 ==> Bond4 = 4$$

$$Bond3 ==> 10,400(Bond3) = 0 ==> Bond3 = 0$$

$$Bond2 ==> 400(Bond3) + 1080(Bond2) = 250,000$$

$$==> 400(0) + 1080(Bond2) = 250,000 ==> Bond2 = \frac{250,000}{1080} = 231.48148$$

$$Bond1 ==> 400(Bond3) + 80(Bond2) + 1300(Bond1) = 100,000$$

$$==> 400(0) + 80(231.48148) + 1300(Bond1) = 100,000$$

$$Bond1 = \frac{100,000 - 80(231.48148)}{1300} = 62.678$$

Sally wants to fully immunize a future payment of X at time Y using the following two bonds:

a. Bond A is a zero coupon bond maturing in 4 years; and
b. Bond B is a zero coupon bond maturing in 17 years.

Sally pays 231,933.38 for Bond A and 77,311.13 for Bond B.

Determine X and Y if the annual effective interest rate of 7%.

Solution:

PV of Assets = PV of Liabilities = 231,933.38+77,311.13 = 309,244.21

Duration of Liability is Y. It must equal duration of Assets.

$$==> \frac{4(213,933.38)+(17)(77,311.13)}{309,244.21} = Y ==> Y = 7.25$$

PV of Assets = PV of Liabilities $==> 309,244.21 = X(1.07)^{-7.25}$

$$==> X = 505,050$$

Allison Kelly & Company must pay 300,000 to Emily at the end of 13 years. The Company wants to use Reddington Immunization to protect itself from interest rate changes. The company will use the following two bonds to immunize itself at an annual effective interest rate of 5%:

a. Bond 1 is a zero coupon bond that matures for 10,000 at the end of 8 years.

b. Bond 2 is a zero coupon bond that matures for 10,000 at the end of 20 years.

Assuming Allison Kelly can buy any number of the above bonds including partial bonds, determine the number of Bond 2 that should be purchased.

Solution:

First we must find the amount spent on each bond and then calculate the amount of each bond.

$$\text{Amount Spend on Bond 2} = \frac{D^L - D^1}{D^2 - D^1}\left[PV \text{ of Liability}\right]$$

$$= \frac{13-8}{20-8}\left[300,000(1.05)^{-13}\right] = 66,290.17$$

$$\text{Price of Bond 2} = (10,000)(1.05)^{-20} = 3768.89483$$

$$\text{Number of Bond 2} = \frac{\text{Amount Spent}}{\text{Price}} = \frac{66,290.17}{3768.89483} = 17.59$$

The Bowman Insurance Company will pay Katie 250,000 at the end of each year for the next four years. Bowman wants to exactly match the payments using the following four bonds:

a. Bond 1 is a one year bond with annual coupons of 400 and a maturity value of 10,000.

b. Bond 2 is a two year bond with annual coupons of 500 and a maturity value of 5000.

c. Bond 3 is a three year bond with annual coupons of 200 and a maturity value of 2300.

d. Bond 4 is a zero coupon bond maturing in four years with a maturity value of 20,000.

Determine the number of Bond 1 which Bowman should purchase.

Solution:

$$\text{Time 1==>}(Bond1)(10,400)+(Bond2)(500)+(Bond3)(200)+(Bond4)(0)=250,000$$
$$\text{Time 2==>}(Bond1)(0)+(Bond2)(5500)+(Bond3)(200)+(Bond4)(0)=250,000$$
$$\text{Time 3==>}(Bond1)(0)+(Bond2)(0)+(Bond3)(2500)+(Bond4)(0)=250,000$$
$$\text{Time 4==>}(Bond1)(0)+(Bond2)(0)+(Bond3)(0)+(Bond4)(20,000)=250,000$$

$$\text{Using Time 3==>}Bond3=\frac{250,000}{2500}=100$$

$$\text{Using Time 2==>}Bond2=\frac{250,000-(100)(200)}{5500}=41.81818$$

$$\text{Using Time 1==>}Bond1=\frac{250,000-(100)(200)-(41.81818)(500)}{10,400}=20.1049$$

Maxwell Company agrees to pay Chloe 850,000 at the end of 10 years. Chrissy, who is the Chief Actuary for Maxwell Company, wants to use Full Immunization with the following two bonds:

a. Bond A is a zero coupon bond which matures for 10,000 at the end of 6 years.

b. Bond B is a zero coupon bond which matures for 12,000 at the end of 13 years.

Maxwell Company can purchase any number of each bond including partial bonds.

At an annual effective interest rate of 9%, determine the number of each bond that Maxwell Company should purchase.

Solution:

$$AmountBondA = \frac{D^B - D^L}{D^B - D^A}(PV\ \ of\ \ Liability) = \left(\frac{13-10}{13-6}\right)(850,000)(1.09)^{-10} = 153,878.22$$

$$NumberBondA = \frac{AmountBondA}{PriceBondA} = \frac{153,878.22}{(10,000)(1.09)^{-6}} = 25.807$$

$$AmountBondB = \frac{D^L - D^A}{D^B - D^A}(PV\ \ of\ \ Liability) = \left(\frac{10-6}{13-6}\right)(850,000)(1.09)^{-10} = 205,170.96$$

$$NumberBondB = \frac{AmountBondB}{PriceBondB} = \frac{205,170.96}{(12,000)(1.09)^{-13}} = 52.418$$

The Purdue Insurance Company has agreed to pay Yuchen 50,000 at the end of one year, 100,000 at the end of two years, and 200,000 at the end of 3 years.

Purdue Insurance Company wants to absolutely match the payments of Yuchen using the following three bonds:

a. Bond A is a zero coupon bond that matures in one year for 10,000.

b. Bond B is a two year bond with annual coupons of 100 and a maturity value of 1300.

c. Bond C is a three year bond with annual coupons of 400 and a maturity value of 5000.

Calculate the number of each bond that Purdue should purchase. Assume that partial bonds can be purchased.

Solution:

$$5400C = 200,000 ==> C = 37.037$$

$$400C + 1400B = 100,000 ==> 1400B = 100,000 - (400)(37.037) ==> B = 60.847$$

$$400C + 100B + 10,000A = 50,000$$

$$==> 10,000A = 50,000 - (400)(37.037) - 100(60.847) ==> A = 2.910$$

Stanley Insurance Company has agreed to pay Jacqueline 400,000 at the end of each year for three years. Stanley wants to use the following three bonds to exactly match the payments to Jacqueline.

a. Bond A is a one year bond with a maturity value of 1000, annual coupons of 70, and a price of 1005.

b. Bond B is a two year bond with annual coupons of 200 and a maturity value of 1000. It sells to yield an annual effective interest rate of 8%.

c. Bond C is a three year bond with annual coupons of 270. The maturity value and the price of this bond is 3000.

Calculate the cost to buy the bonds to exactly match the payments. Assume that we can purchase partial bonds.

Solution:

Time	**Year 1**	**Year 2**	**Year 3**
Payments	400,000	400,000	400,000
Bond A	1070	0	0
Bond B	200	1200	0
Bond C	270	270	3270

Let A be the number of Bond A purchased. Let B be the number of Bond B purchased. Let C be the number of Bond C purchased.

$$3270(C) = 400,000 ==> C = \frac{400,000}{3270} = 122.3242$$

$$1200(B) + 270(C) = 400,000 ==> B = \frac{400,000 - (270)(122.3242)}{1200} = 305.8104$$

$$1070(A) + 200(B) + 270(C) = 400,000$$

$$A = \frac{400,000 - 200(305.8104) - 270(122.3242)}{1070} = 285.8041$$

$$Total\ \text{Cos}\,t = (A)(1005) + (B)(\text{Price of B}) + (C)(3000)$$

$$\boxed{\text{Price of B} = 200a_{\overline{2|}} + 1000v^2 = 200\left(\frac{1-(1.08)^{-2}}{0.08}\right) + 1000(1.08)^{-2} = 1213.99}$$

$$= (285.8041)(1005) + (305.8104)(1213.99) + (122.3242)(3000) = 1,025,456.49$$

Billie has promised to make a payment of 1,000,000 to Bokun at the end of 7.5 years. Billie wants to immunize this payment using Reddington Immunization and the following two zero coupon bonds:

a. Bond 1 is a zero coupon bond with a maturity value of 10,000 at the end of 4 years.

b. Bond 2 is a zero coupon bond with a maturity value of 15,000 at the end of 8 years.

Assuming an interest rate of 8%, determine the number of Bond 1 which Billie should purchase. Assume that we can purchase partial bonds.

Solution:

We can use the shortcut to do this problem.

$$\text{Amount of Money to be used to Purchase Bond 1} = \left[\frac{D_{Bond\,2} - D_{Liability}}{D_{Bond\,2} - D_{Bond\,1}}\right](PVofLiability)$$

$$= \left[\frac{8-7.5}{8-4}\right](1,000,000)(1.08)^{-7.5} = 70,182.98683$$

$$\text{Price of Bond 1} = (10,000)(1.08)^{-4} = 7350.298527$$

$$\text{Number of Bonds} = \frac{\text{Amount of Money}}{\text{Price of Bond 1}} = \frac{70,182.98683}{7350.298527} = 9.5483$$