



You are given the following spot interest rates:

| $t$ | $r_t$ | $t$ | $r_t$ |
|-----|-------|-----|-------|
| 0.5 | 5.00% | 3.0 | 6.60% |
| 1.0 | 5.40% | 3.5 | 6.85% |
| 1.5 | 5.75% | 4.0 | 7.05% |
| 2.0 | 6.05% | 4.5 | 7.20% |
| 2.5 | 6.35% | 5.0 | 7.30% |

Mueller Corporation and Kellyn Bank enter into an interest rate swap. The swap is a five year swap with annual periods. However, the swap is a deferred swap and there is no swap of interest rates during the first three years. The swap has a notational amount of 1,300,000 during the fourth year and 900,000 during the fifth year.

Determine the swap interest rate to five decimal places under this swap.

**Solution:**

$$R = \frac{\sum Q \cdot f \cdot P}{\sum Q \cdot P}$$

$$(1.066)^3(1 + f_{[3,4]}) = (1.0705)^4 \Rightarrow f_{[3,4]} = \frac{(1.0705)^4}{(1.066)^3} - 1 = 0.084114299$$

$$(1.0705)^4(1 + f_{[4,5]}) = (1.0730)^5 \Rightarrow f_{[4,5]} = \frac{(1.0730)^5}{(1.0705)^4} - 1 = 0.08305852$$

$$R = \frac{(1,300,000)(0.084114299)(1.0705)^{-4} + (900,000)(0.08305852)(1.0730)^{-5}}{(1,300,000)(1.0705)^{-4} + (900,000)(1.0730)^{-5}}$$

$$= 0.083702596 \approx 0.08370$$

Zach and Alex enter into a five year interest rate swap with annual swap period. Under the swap, Zach agrees to pay the fixed interest rate of 6.42% to Alex. In return, Alex agrees to pay the variable interest rate to Zach at the end of each year. The amount of the swap is 8,000,000.

At the beginning of the second year of the swap, the variable interest rate is 5.79%.

Determine the net swap payment at the end of the second year and state who makes that payment.

**Solution:**

Zach will pay fixed rate  $\Rightarrow (8,000,000)(0.0642) = 513,600$

Alex will pay variable rate  $\Rightarrow (8,000,000)(0.0579) = 463,200$

Net Swap Payment:  $513,600 - 463,200 = 50,400$

Zach will make a payment of 50,400 to Alex.

You are given the following spot interest rates and information for the following questions:

| $t$ | $r_t$ | $t$ | $r_t$ |
|-----|-------|-----|-------|
| 0.5 | 4.00% | 3.0 | 5.85% |
| 1.0 | 4.50% | 3.5 | 6.20% |
| 1.5 | 4.95% | 4.0 | 6.35% |
| 2.0 | 5.30% | 4.5 | 6.45% |
| 2.5 | 5.60% | 5.0 | 6.50% |

Jinks Corporation has a four year loan from Anderson Bank for an amount of 400,000. The loan has a variable interest rate which is equal to the one year spot rate at the beginning of each year of the loan. Jinks will pay the interest on the loan at the end of each year for four years. Additionally, Jinks will pay the principal of the loan which is 400,000 at the end of the fourth year.

Jinks would like to have a fixed interest rate instead of a variable interest rate. Therefore, Jinks enters into an interest rate swap with Lai Investment Bank. Under the swap, Jinks will pay a fixed interest rate to Lai and Lai will pay the variable interest rate to Jinks. The terms of the swap mirror the terms of the loan.

1. Answer the following four questions which will count as one question on the exam:

a. Who is the receiver in this scenario?

**Lai Investment Bank**

b. Who is the payer in this scenario?

**Jinks Corporation**

c. State the Settlement Period for this Swap.

**One Year**

d. State the Notional Amount for this Swap.

**400,000**

2. Determine the Swap Rate for this Swap.

$$R = \frac{1 - P_4}{P_1 + P_2 + P_3 + P_4} = \frac{1 - (1.0635)^{-4}}{(1.045)^{-1} + (1.053)^{-2} + (1.0585)^{-3} + (1.0635)^{-4}} = 0.06266$$

You are given the following spot interest rates:

| $t$ | $r_t$ | $t$ | $r_t$ |
|-----|-------|-----|-------|
| 0.5 | 4.00% | 3.0 | 5.85% |
| 1.0 | 4.50% | 3.5 | 6.20% |
| 1.5 | 4.95% | 4.0 | 6.35% |
| 2.0 | 5.30% | 4.5 | 6.45% |
| 2.5 | 5.60% | 5.0 | 6.50% |

Shina buys a deferred interest rate swap. The Swap Tenor is five years. There is no interest rate swap for the first three years. During the last two years, Shina swaps a variable interest rate for a fixed interest rate. The variable interest rate is equal to the one year spot rate at the beginning of each year. The swap is based on a notional amount of 500,000 during the 4<sup>th</sup> year and 1,000,000 during the last year.

Determine the Swap Rate.

**Solution:**

$$R = \frac{\sum Q \cdot f \cdot P}{\sum Q \cdot P}$$

$$(1.0585)^3(1 + f_{[3,4]}) = (1.0635)^4 \implies f_{[3,4]} = \frac{(1.0635)^4}{(1.0585)^3} - 1 = 0.078642157$$

$$(1.0635)^4(1 + f_{[4,5]}) = (1.0650)^5 \implies f_{[4,5]} = \frac{(1.065)^5}{(1.0635)^4} - 1 = 0.071021186$$

$$R = \frac{(500,000)(0.078642157)(1.0635)^{-4} + (1,000,000)(0.071021186)(1.065)^{-5}}{(500,000)(1.0635)^{-4} + (1,000,000)(1.065)^{-5}} = 0.073689$$

The following information is provided for the next two questions:

On January 1, **2015**, Joe entered into a five year interest rate swap with swap periods of one year. The swap has a notional amount of 300,000. Under the swap, Joe will pay a fixed rate of 6.5% at the end of each year and will receive a payment based on the variable interest rate which is equal to the one year spot rate at the beginning of each year.

On January 1, **2018**, the spot interest rates are as follows:

| $t$ | $r_t$ |
|-----|-------|
| 0.5 | 5.80% |
| 1.0 | 6.20% |
| 1.5 | 6.75% |
| 2.0 | 7.00% |
| 2.5 | 7.20% |

1. Calculate the net swap payment for Joe at the end of the fourth year of the Swap. (Be sure to state whether Joe will make the payment or receive the payment.)

**Solution:**

Joe will pay fixed rate  $\implies (300,000)(0.065) = 19,500$

Joe will receive variable rate  $\implies (300,000)(0.062) = 18,600$

$$19,500 - 18,600 = 900$$

Joe will make a payment of 900.

2. Joe decides to sell the swap at the start of the fourth year of the swap.

Determine the market value of the swap from Joe's viewpoint.

**Solution:**

Joe will pay fixed rate  $\implies (300,000)(0.065) = 19,500$ . He will pay this at the end of the fourth year and at the end of the fifth year.

Joe will receive variable rate  $\implies (300,000)(0.062) = 18,600$  at the end of the fourth year. He will also receive the variable rate at the end of the fifth year. We base that

calculation on the forward rate  $= \frac{(1.07)^2}{1.062} - 1 = 0.07806$

$\implies (300,000)(0.07806) = 23,418$  is the amount Joe will receive at the end of 5th year.

Market Value = Present Value that Joe will receive less Present Value that Joe will pay

$$= \frac{18,600 - 19,500}{1.062} + \frac{23,418 - 19,500}{(1.07)^2} = 2574.68$$

You are given the following spot interest rates and information for the following questions:

| $t$ | $r_t$ | $t$ | $r_t$ |
|-----|-------|-----|-------|
| 0.5 | 3.00% | 3.0 | 4.85% |
| 1.0 | 3.50% | 3.5 | 5.20% |
| 1.5 | 3.95% | 4.0 | 5.35% |
| 2.0 | 4.30% | 4.5 | 5.45% |
| 2.5 | 4.60% | 5.0 | 5.50% |

Chen Corporation has a three year loan from Luo Bank for an amount of 500,000. The loan has a variable interest rate which is equal to the one year spot rate at the beginning of each year of the loan. Chen will pay the interest on the loan at the end of each year for three years. Additionally, Chen will pay the principal of the loan which is 500,000 at the end of the third year.

Chen would like to have a fixed interest rate instead of a variable interest rate. Therefore, Chen enters into an interest rate swap with Zhu Investment Bank. Under the swap, Chen will pay a fixed interest rate to Zhu and Zhu will pay the variable interest rate to Chen. The terms of the swap mirror the terms of the loan.

1. Answer the following three questions which will count as one question for this exam.

a. State the counterparties to the interest rate swap.

**Chen is the payer and Zhu is the receiver.**

b. State the tenor of the swap.

**Three Years**

c. State the notional amount of the swap.

**500,000**

2. Determine the fixed interest rate under this swap.

**Solution:**

$$R = \frac{1 - P_3}{P_1 + P_2 + P_3} = \frac{1 - (1.0485)^{-3}}{(1.035)^{-1} + (1.043)^{-2} + (1.0485)^{-3}} = 0.04811$$



You are given the following information for following questions:

Sadi and Yash enter into a five year interest rate swap with annual swap payments. Under the swap, Sadi will pay a fixed interest rate of 6.4% to Yash and Yash will pay the variable interest rate to Sadi. The variable interest rate will be the one year spot interest rate at the beginning of each year.

The notional amount of the swap is 100,000.

At the end of three years, there are two years left on the swap. The spot interest rates at the end of three years are:

| $t$ | $r_t$ |
|-----|-------|
| 1   | 0.060 |
| 2   | 0.067 |
| 3   | 0.072 |
| 4   | 0.075 |
| 5   | 0.077 |

1. Determine the net swap payment in the fourth year of the swap. State who pays this payment.

**Solution:**

**Sadi owes  $(100,000)(0.064)=6400$**

**Yash owes  $(100,000)(0.060)=6000$**

**Sadi Pays Yash 400**

2. If at the end of the third year Sadi decides to sell the swap to Daniel, determine the market value of the swap (from Sadi's viewpoint) at the end of three years.

**Solution:**

Market Value = *PV of Projected Cashflows*

$$f_{[1,2]} = \frac{(1.067)^2}{1.06} - 1 = 0.074046226$$

$$= \frac{100,000(0.06 - 0.064)}{1.06} + \frac{100,000(0.074046226 - 0.064)}{(1.067)^2} = 505.06$$

You are given the following spot interest rates:

| $t$ | $r_t$ |
|-----|-------|
| 1.0 | 3.50% |
| 2.0 | 4.30% |
| 3.0 | 4.85% |
| 4.0 | 5.35% |
| 5.0 | 5.50% |

Andrew and Michael enter into a three year interest rate swap. This is a deferred swap as there are no swap payments at the end of the first year. Annual swap payments will be made at the end of the last two years. Andrew will be the payer and Michael will be the receiver under the swap. The variable interest rate will be the one year spot interest rate at the beginning of each year.

The notional amount of the swap during the second year is 200,000. The notional amount of the swap during the third year is 400,000.

Determine the fixed interest rate under this swap and state whether Andrew or Michael will be paying this interest rate.

**Solutions:**

Andrew pays the fixed rate.

$$f_{[1,2]} = \frac{(1.043)^2}{1.035} - 1 = 0.051061836$$

$$f_{[2,3]} = \frac{(1.0485)^3}{(1.043)^2} - 1 = 0.059587162$$

$$\begin{aligned}
 R &= \frac{200,000 f_{[1,2]} (1+r_2)^{-2} + 400,000 f_{[2,3]} (1+r_3)^{-3}}{200,000 (1+r_2)^{-2} + 400,000 (1+r_3)^{-3}} \\
 &= \frac{200,000 (0.051061836) (1.043)^{-2} + 400,000 (0.059587162) (1.0485)^{-3}}{200,000 (1.043)^{-2} + 400,000 (1.0485)^{-3}} \\
 &= 0.056635
 \end{aligned}$$

NULL INC has a 1,000,000 loan from Zajac Bank. Under the terms of the loan, NULL will pay interest annually to Zajac Bank based on LIBOR plus 120 basis points. Additionally, NULL will pay the principal of 1,000,000 at the end of ten years.

NULL would prefer to know the annual interest cost that will be incurred. To fix the interest rate on the loan, NULL enters into a ten-year interest rate swap with a notional amount of 1,000,000 and annual settlement dates. The terms of the swap are that NULL will swap a variable rate of LIBOR plus 70 basis points for a fixed rate of 6.2%.

During the third year of the loan, LIBOR is 5.7%.

Determine the net interest payment that NULL will make during the third year.

**Solution:**

Payment to the Bank

$$1,000,000(0.057 + 0.012) = 69,000$$

Payment from the Swap

$$1,000,000(0.057 + 0.007 - 0.062) = 2000$$

$$\text{Net Interest Payment} = 69,000 - 2000 = 67,000$$

You are given the following spot interest rates and information for the questions below:

| Time t | Spot Rate $r_t$ |
|--------|-----------------|
| 1      | 0.053           |
| 2      | 0.057           |
| 3      | 0.063           |
| 4      | 0.071           |
| 5      | 0.080           |

The F&G Corporation borrows 700,000 from Abbott Bank. The terms of the loan are such that F&G will pay a variable interest rate on the loan each year and will repay the 700,000 at the end of four years. The variable interest rate will be the one year spot rate at the beginning of each settlement period.

F&G is uncomfortable with the risk of having a variable loan interest rate. Therefore, F&G purchases a four year interest rate swap from M&J Investment Bank. At the end of each year, F&G will pay a fixed rate to M&J Investment Bank. At the end of each year, M&J will pay the variable rate to F&G. The amount of the payment will be based on the amount of the loan of 700,000.

1. Answer the questions below. These four parts are worth one question.

a. List the Payer under this agreement.

**F&G Corporation**

b. List the Receiver under this agreement.

**M&J Investment Bank**

c. State the Settlement Period under the agreement.

**The settlement period is one year.**

d. List the notional amount in the first year under this agreement.

**The notional amount in the first year is 700,000.**

2. Calculate the swap rate under this agreement.

**Solution:**

$$R = \frac{1 - P_4}{P_1 + P_2 + P_3 + P_4} = \frac{1 - (1.071)^{-4}}{(1.053)^{-1} + (1.057)^{-2} + (1.063)^{-3} + (1.071)^{-4}} = 0.069807645$$

You are given the following spot interest rates and information for the following questions:

| Time t | Spot Rate $r_t$ |
|--------|-----------------|
| 1      | 0.053           |
| 2      | 0.057           |
| 3      | 0.063           |
| 4      | 0.071           |
| 5      | 0.080           |

The F&G Corporation borrows 700,000 from Abbott Bank. The terms of the loan are such that F&G will pay a variable interest rate on the loan each year and will repay the 700,000 at the end of four years. The variable interest rate will be the one year spot rate at the beginning of each settlement period.

F&G is uncomfortable with the risk of having a variable loan interest rate. Therefore, F&G purchases a four year interest rate swap from M&J Investment Bank. At the end of each year, F&G will pay a fixed rate to M&J Investment Bank. At the end of each year, M&J will pay the variable rate to F&G. The amount of the payment will be based on the amount of the loan of 700,000.

- Based on the spot interest rate curve at the time of the loan, what is the implied rate that F&G would pay to Abbott Bank in the third year of the loan?

**Solution:**

The implied rate is  $f_{[2,3]}$ .

$$(1 + r_3)^3 = (1 + r_2)^2(1 + f_{[2,3]}) \implies f_{[2,3]} = \frac{(1 + r_3)^3}{(1 + r_2)^2} - 1 = \frac{(1.063)^3}{(1.057)^2} - 1 = 0.075102$$

- Calculate the net swap payment at the end of the first year. State who receives the payment and who makes the payment.

**Solution:**

$$\text{F\&G Owes (notional amount)(fixed rate)} = (700,000)(0.069807645) = 48,865.35$$

$$\text{M\&J Owes (notional amount)(variable rate)} = (700,000)(0.053) = 37,100.00$$

F&G Pays 11,765.35 to M&J

You are given the following spot interest rates:

| Time t | Spot Rate $r_t$ |
|--------|-----------------|
| 1      | 0.053           |
| 2      | 0.057           |
| 3      | 0.063           |
| 4      | 0.071           |
| 5      | 0.080           |

Thomas and Brett enter into a five year deferred interest rate swap. There will be no swap during the first three years. The settlement periods will be one year. Under the swap, Thomas agrees to pay a variable interest rate to Brett at the end of the fourth year and at the end of the fifth year. The variable interest rate is based on the one year spot interest rate at the start of each year.

In return, Brett agrees to pay a fixed rate to Thomas at the end of the fourth year and the fifth year.

The notional swap amount is 100,000 for both settlement periods.

Calculate the swap rate for this swap.

**Solution:**

$$R = \frac{P_3 - P_5}{P_4 + P_5} = \frac{(1.063)^{-3} - (1.080)^{-5}}{(1.071)^{-4} + (1.080)^{-5}} = 0.10547$$

The current spot interest rate curve is as follows:

| $t$  | $r_t$ |  | $t$  | $r_t$ |
|------|-------|--|------|-------|
| 0.25 | 2.50% |  | 1.75 | 3.40% |
| 0.50 | 2.65% |  | 2.00 | 3.48% |
| 0.75 | 2.79% |  | 2.25 | 3.80% |
| 1.00 | 2.92% |  | 2.50 | 4.10% |
| 1.25 | 3.10% |  | 2.75 | 4.35% |
| 1.50 | 3.25% |  | 3.00 | 4.50% |

Cai has a two year loan for 500,000 which has a variable interest rate that resets at the beginning of each six month period. The interest rate will be the six month spot interest rate at the beginning of each six month period.

Cai enters into an interest rate swap where she is the payer. The characteristics of the swap mirror those of the loan.

Determine the six month swap rate that Cai will pay.

**Solution:**

$$R = \frac{1 - P_2}{P_{0.5} + P_1 + P_{1.5} + P_2} = \frac{1 - (1.0348)^{-2}}{(1.0265)^{-0.5} + (1.0292)^{-1} + (1.0325)^{-1.5} + (1.0348)^{-2}} = 0.0171956$$

Sue and Gavin enter into a three year swap. Under the swap, Sue is the payer and Gavin is the receiver. The swap has annual settlement periods. The variable rate to be paid is the one year spot rate at the beginning of each year for the next three years.

The following is the spot interest rate curve:

| Time t | Spot Rate $r_t$ |
|--------|-----------------|
| 1      | 0.060           |
| 2      | 0.055           |
| 3      | 0.050           |
| 4      | 0.048           |
| 5      | 0.046           |

The notional amount for the swap changes each year. The notional amount during the first year is 500,000. The notional amount during the second year is 300,000. The notional amount during the third year is 100,000.

Calculate the swap rate for this swap.

**Solution:**

$$R = \frac{Q_1 \cdot f_{[0,1]}^* \cdot P_1 + Q_2 \cdot f_{[1,2]}^* \cdot P_2 + Q_3 \cdot f_{[2,3]}^* \cdot P_3}{Q_1 \cdot P_1 + Q_2 \cdot P_2 + Q_3 \cdot P_3}$$

|  |
|--|
| $f_{[0,1]} = r_1 = 0.06$ $f_{[1,2]} = \frac{(1.055)^2}{1.06} - 1 = 0.050023585$ $f_{[2,3]} = \frac{(1.05)^3}{(1.055)^2} - 1 = 0.040070978$ |
|--|

$$= \frac{(500,000)(0.06)(1.06)^{-1} + (300,000)(0.050023585)(1.055)^{-2} + (100,000)(0.040070978)(1.05)^{-3}}{(500,000)(1.06)^{-1} + (300,000)(1.055)^{-2} + (100,000)(1.05)^{-3}}$$

$$= 0.054671$$