Topic: Loan Amortization

Josh has a 10 year loan which is being repaid with non-level annual payments. The first payment is 10,000. The second payment is 9000. The third payment is 8000. The payments continue to decrease until the last payment is 1000.

The loan has an annual effective interest rate of 6%.

Determine the principal in the payment of 3000 at the end of the 8th year.

Solution:

To find the principal in the 8th payment, we need to find the OLB after the 7th payment.

$$OLB_7 = 3000(1.06)^{-1} + 2000(1.06)^{-2} + 1000(1.06)^{-3}$$

$$= 5449.80$$

$$I_8 = (5449.80)(0.06) = 326.99$$

$$P_8 = PMT_8 - I_8 = 3000 - 326.99 = 2673.01$$

A 30 year mortgage loan is being repaid with level monthly payments of 1630.48. The principal in the 90^{th} payment is 352.27.

Determine the **annual effective interest rate** on the loan.

Solution:

$$\text{Principal in } 90^{\text{th}} \text{ Payment } = P_{90} = 352.27$$

$$P_k = Qv^{n-k+1}$$

$$n = (30)(12) = 360$$

$$P_{90} = 1630.48v^{360-90+1} = 1630.48v^{271}$$

$$= 1630.48\left(1+\frac{i^{(12)}}{12}\right)^{-271} = 352.27$$

$$\Rightarrow \left(1+\frac{i^{(12)}}{12}\right)^{-271} = \frac{352.27}{1630.48} = 0.216052941$$

$$\Rightarrow \frac{i^{(12)}}{12} = (0.216052941)^{-1/271} - 1 = 0.00567$$

$$\text{Annual Effective Interest Rate } = i$$

$$i = \left(1+\frac{i^{(12)}}{12}\right)^{12} - 1 = (1.00567)^{12} - 1 = 0.070202521 \approx 0.07020$$

Shobana takes out a 30 year mortgage loan to buy a new house. The loan is for 1,000,000 and will be repaid with level monthly payments. The interest rate is 9% compounded monthly.

Determine the principal in the 240th payment.

Solution:

The best way to do this is with the calculator

$$\boxed{N} \leftarrow (30)(12) = 360$$

$$\boxed{I/Y} \leftarrow \frac{0.09}{12}(100) = 0.75$$

$$\boxed{PV} \leftarrow -1{,}000{,}000$$

$$\boxed{CPT}\boxed{PMT} \Rightarrow 8{,}046.226169$$

$$\boxed{2nd}\boxed{Amort}$$

$$\boxed{P1} \leftarrow 240$$

$$\boxed{P2} \leftarrow 240$$

$$\boxed{\downarrow}\boxed{\downarrow}\,\mathrm{Pr}\,in \text{ ==> } 3257.92$$

James has a loan for 100,000 which will be repaid with level annual payments of Q for n years. The interest rate on the loan is 5.43%.

The principal in the 20^{th} payment is 60.90.

Determine Q .

Solution:

Interest in the First Payment = (L)(i) = (100,000)(0.0543) = 5430

Principle in the first payment = (principle in the 20th Payment)$(1.0543)^{-(20-1)}$

$= 60.90(1.0543)^{-19} = 22.30$

$Q = 5430 + 22.30 = 5452.30$

Ben has a loan for 50,000 which will be repaid with three non-level annual payments. The interest rate on the loan is an annual effective interest rate of 10%.

The first loan repayment is 30,000. The second loan repayment 20,000. The third loan repayment is P. The loan is repaid after the third payment.

Calculate the principal in the second payment.

Solution:

k	Payment	Interest	Principal	OLB
0				50,000
1	30,000	(50,000)(0.10)=5000	30,000-5000=25,000	50,000-25,000=25,000
2	20,000	(25,000)(0.10)=2500	20,000-2500=17,500	

David has a loan with n annual payments of 1000. The interest in the 15th payment is 799.02. The principal in the 20th payment is 246.89.

Find the outstanding loan balance right after the 30th payment.

Solution:

$$P_{15} = 1000 - 799.02 = 200.98$$

$$P_{15}(1+i)^{20-15} = P_{20} ==> 200.98(1+i)^5 = 246.89 ==> i = \left(\frac{246.89}{200.98}\right)^{01/5} - 1 = 0.042$$

$$OLB_{30}(0.042) = I_{31}$$

$$P_{31} = (246.89)(1.042)^{11} = 388.19 ==> I_{31} = 1000 - 388.19 = 611.81$$

$$OLB_{30} = \frac{I_{31}}{0.042} = \frac{611.81}{0.042} = 14,566.90$$

There are other ways to do this which will get you a slightly different answer.

A loan which is being repaid with level annual payments at an interest rate of 8%. The interest in the 10th payment is 846.41. The principal in the 20th payment is 609.92.

Calculate the amount of the loan.

Solution:

$$P_{10} = P_{20}(1.08)^{-10} = 609.92(1.08)^{-10} = 282.51$$

$$Q = P_{10} + I_{10} = 846.41 + 282.51 = 1128.92$$

$$P_1 = 282.51(1.08)^{-9} = 141.326$$

$$I_1 = 1128.92 - 141.326 = 987.594$$

$$I_1 = iL \text{ ==> } 987.594 = (0.08)(L) \text{ ==> } L = 12,344.93 \approx 12,345$$

Aisling has a 30 year loan with non-level annual payments. The payment at the end of each odd numbered year (year 1, 3, 5, …, 29) will be 5000. The payment at the end of each even numbered year (year 2, 4, 6, …, 30) will be 10,000.

The interest rate on the loan is an annual effective interest rate of 6%.

Determine the principal in the payment of 5000 at the end of the 27th year.

Solution:

To find the principle in the 27th payment, we need to find the OLB after the 26th payment.

$$OLB_{26} = 5000(1.06)^{-1} + 10,000(1.06)^{-2} + 5000(1.06)^{-3} + 10,000(1.06)^{-4} = 25,735.98$$

$$I_{27} = (25,735.98)(0.06) = 1544.16$$

$$P_{27} = 5000 - 1544.16 = 3455.84$$

Bokun has a loan that is being repaid with n annual payments of 1000.

The principal in the 6th payment is 147.53. The principal in the 12th payment is 206.80.

Calculate the interest in the 20th payment.

Solution:

$$147.53(1+i)^{12-6} = 206.80 \Longrightarrow (1+i)^6 = \frac{206.80}{147.53} \Longrightarrow i = \left(\frac{206.80}{147.53}\right)^{1/6} - 1 = 0.0579$$

$$\text{Prin}_{20} = 206.80(1.0579)^{20-12} = 324.42$$

$$Int_{20} = Payment - \text{Prin}_{20} = 1000 - 324.42 = 675.58$$

Christian has a 20 year loan of 110,950.62 that is being repaid with annual payments. The first payment is 1000. The second payment is 2000. The third payment is 3000. The payments continue to increase in the same pattern with a payment of 20,000 being made at the end of the 20^{th} year.

The interest rate on the loan is an annual effective interest rate of 5%.

Calculate the principal in the third payment.

Solution:

We must find the Outstanding Loan Balance after two payments.

$OLB_2 = \text{Accumulated Value of Past Cash Flows}$

$= (110{,}950.62)(1.05)^2 - (1000)(1.05) - 2000 = 119{,}273.06$

$Int_3 = (OLB_2)\,i = (119{,}273.06)(0.05) = 5963.65$

$\text{Prin}_3 = \text{Payment} - Int_3 = 3000 - 5963.65 = -2963.65$

A 20 year loan is being repaid with 20 annual non-level payments. The first payment is 25,000. The second payment is 24,000. The payments continue to decrease until the last payment of 6000 is paid. The interest rate on the loan is an annual effective rate of 6%.

Calculate the principal in the 11th payment.

Solution:

We want to find the outstanding loan balance at time 10.

$$\text{OLB}_{10} = (15,000)a_{\overline{10|}} - \left(\frac{1000}{0.06}\right)\left(a_{\overline{10|}} - 10(1.06)^{-10}\right) = 80,798.98$$

$$I_{11} = \left(OLB_{10}\right)(0.06) = (80,798.98)(0.06) = 4847.94$$

$$P_{11} = 15,000 - 4847.94 = 10,152.06$$

Yuchen has a loan which is being repaid with level monthly payments of 500. The interest in the 30th payment is 113.74. The principal in the 36th payment is 402.77.

Determine the amount of the loan.

Solution:

$$\text{Principal in 30th Payment} = P_{30} = 500 - 113.74 = 386.26$$

$$P_{30}(1+i)^6 = P_{36} \text{ ==> } 386.26(1+i)^6 = 402.77 \text{ ==> } i = \left(\frac{402.77}{386.26}\right)^{\frac{1}{6}} - 1 = 0.007$$

$$P_1 = P_{30}(1+i)^{1-30} \text{ ==> } P_1 = 386.26(1.007)^{-29} = 315.52$$

$$I_1 = 500 - 315.52 = 184.48 = OLB_0(i) = OLB_0(0.007)$$

$$\text{Amount of Loan} = OLB_0 = \frac{184.48}{0.007} = 26{,}354.29$$

If you carry more decimals or do the problem a different way, your answer will be slightly different.

Spencer is repaying a loan with n level annual payments of 8726.18. The interest in the fifth payment is 4945.11. The interest in the 10th payment is 4014.28.

Determine n .

Solution:

$$\text{Prin}_5 = 8726.18 - 4945.11 = 3781.07$$

$$\text{Prin}_{10} = 8726.18 - 4014.28 = 4711.90$$

$$\text{Prin}_5 (1.05)^5 = \text{Prin}_{10} \text{ ==> } 3781.07(1+i)^5 = 4711.90 \text{ ==> } i = \left(\frac{4711.90}{3781.07}\right)^{1/5} - 1 = 0.045$$

$$Qv^{n-k+1} = \text{Prin}_k$$

$$8726.18\left(\frac{1}{1.045}\right)^{n-5+1} = 3781.07$$

$$\left(\frac{1}{1.045}\right)^{n-4} = \frac{3781.07}{8726.18} \text{ ==> } (n-4)\left[\ln\left(\frac{1}{1.045}\right)\right] = \ln\left(\frac{3781.09}{8726.18}\right)$$

$$n = \frac{\ln\left(\frac{3781.09}{8726.18}\right)}{\ln\left(\frac{1}{1.045}\right)} + 4 = 23$$