

**Topic: Net Present Value**

Thomas Corporation invests 10 million to build a new factory. In return, he expects to receive a payment of 6 million at the end of 2 years and 8 million at the end of 4 years.

Determine the Internal Rate of Return on Thomas' investment.

**Solution:**

$$-10,000,000 + 6,000,000(1+i)^{-2} + 8,000,000(1+i)^{-4} = 0$$

Let  $x = (1+i)^{-2}$

$$-10,000,000 + 6,000,000x + 8,000,000x^2 = 0 \implies -10 + 6x + 8x^2 = 0$$

$$x = \frac{-6 + \sqrt{6^2 - 4(8)(-10)}}{2(8)} = 0.80425$$

$$(1+i)^{-2} = 0.80425 \implies i = 0.11508 = 11.508\%$$

**OR**

Using the BA-II+ Calculator:

$$CF0 = -10,000,000$$

$$C01 = 0; F01 = 1$$

$$C02 = 6,000,000; F02 = 1$$

$$C03 = 0; F03 = 1$$

$$C04 = 8,000,000; F04 = 1$$

$$\boxed{CPT} \boxed{IRR} \rightarrow 11.508\%$$

Wagner Corporation invests 1,000,000 today to build a factory. Wagner expects to have the following cash flows:

Time	Cash Flow
1	Negative 400,000
2	Positive 300,000
3	Positive 500,000
4	$X$

With this set of cash flows, Wagner expects to have an Internal Rate of Return of 8%.

Calculate the Net Present Value at 10%.

**Solution:**

$$-1,000,000 - 400,000(1.08)^{-1} + 300,000(1.08)^{-2} + 500,000(1.08)^{-3} + X(1.08)^{-4} = 0$$

$$X = 974,453.76$$

$$NPV = -1,000,000 - 400,000(1.10)^{-1} + 300,000(1.10)^{-2} + 500,000(1.10)^{-3} + 974,453.76(1.10)^{-4}$$

$$= -74,480.05$$

Danny agrees to pay Christine 1000 today. In return, Christine agrees to pay Danny 500 at the end of one year and 700 at the end of two years.

Determine Danny's Internal Rate of Return on the arrangement.

**Solution:**

$$1000(1+i)^2 = 500(1+i) + 700 \implies 10(1+i)^2 - 5(1+i) - 7 = 0$$

$$\text{Using the quadratic } \implies (1+i) = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(10)(-7)}}{(2)(10)} = 1.12321 \implies \boxed{i = 0.12321}$$

*OR*

Using the Calculator

$$\boxed{CF0} \leftarrow -1000; \boxed{C01} \leftarrow 500; \boxed{C02} \leftarrow 700; \boxed{IRR} \boxed{CPT} \rightarrow \boxed{12.321\%}$$

Freija Industries invests 1,000,000 today to build a new plant. Freija expects to earn cash flows of 500,000 at the end of one year and 800,000 at the end of two years.

Based on this investment and these cash flows, calculate Freija's internal rate of return on her investment.

**Solution:**

This problem can be done using your BA II + or using algebra.

Using the BA II+

$$\boxed{CF0} \leftarrow -1,000,000$$

$$\boxed{C01} \leftarrow 500,000$$

$$\boxed{C02} \leftarrow 800,000$$

$$\boxed{IRR} \boxed{CPT} \rightarrow 17.8709\%$$

Using algebra

$$-1,000,000 + (500,000)(1+i)^{-1} + (800,000)(1+i)^{-2} = 0$$

$$\text{Let } x = (1+i)^{-1}$$

$$\implies -1,000,000 + (500,000)x + (800,000)x^2 = 0$$

$$x = (1+i)^{-1} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-500,000 \pm \sqrt{(500,000)^2 - 4(800,000)(-1,000,000)}}{2(800,000)}$$

$$= 0.848385976 \implies i = (0.848385976)^{-1} - 1 = 0.178709$$

Yoon Industries invests  $X$  to build a factory. Yoon expects to receive the following cash flows over the next 4 years.

End of Year	Cash Flow
1	Negative 1 million
2	Positive 2 million
3	Positive 3 million
4	Positive 4 million

At the end of four years, the factory is obsolete and will not generate any further cash flows.

The internal rate of return on this investment is 10%.

Calculate the net present value at 15%.

**Solution:**

$$0 = X - 1(1.10)^{-1} + 2(1.10)^{-2} + 3(1.10)^{-3} + 4(1.10)^{-4}$$

$$X = 1(1.10)^{-1} - 2(1.10)^{-2} - 3(1.10)^{-3} - 4(1.10)^{-4} = -5.729799877 \text{ million}$$

$$NPV = -5.729799877 - 1(1.15)^{-1} + 2(1.15)^{-2} + 3(1.15)^{-3} + 4(1.15)^{-4}$$

$$= -0.82751608 \text{ million} = -827,516.08$$

Zhang LTD is going to build a new factory. Zhang invests 10 million (10,000,000) at time zero. In return for this investment, Zhang expects the following cash flows:

Time t	Cash Flow
1	6 million
2	X million
3	2 million

After three years, the factory will be obsolete and no longer generate cash flows.

The internal rate of return is 10%.

Calculate the net present value at 12%.

**Solution:**

In Millions:

$$-10 + 6(1.1)^{-1} + x(1.1)^{-2} + 2(1.1)^{-3} = 0$$

$$\implies x(1.1)^{-2} = 3.042824944 \implies x = 3.68181818$$

$$NPV = -10 + 6(1.12)^{-1} + 3.68181818(1.12)^{-2} + 2(1.12)^{-3} = -0.28417374 \text{ millions}$$

$$\text{Answer} = -284,173.74$$