Topic: Nominal Interest and Discount Rate

Lauren invests 100,000 in the Diego Fund. The Diego Fund pays a nominal interest rate of 10% compounded quarterly. At the end of Y years, Lauren has 361,111.23.

Alex invests 100,000 in Ethan Bank. Ethan Bank pays simple interest at a rate of s. At the end of Y years, Alex has 361,111.23.

Determine the annual effective interest rate that Alex earns during the 10th year of his investment.

Solution:

Lauren:

$$100,000\left(1+\frac{i^{(4)}}{4}\right)^{(4)(Y)} = 361,111.23 \implies \left(1+\frac{0.10}{4}\right)^{(4)(Y)} = 3.6111123$$

$$(4)(Y)\ln(1.025) = \ln(3.6111123)$$

$$Y = \frac{\ln(3.6111123)}{(4)\ln(1.025)} = 13$$

Alex:

$$100,000(1+13s) = 361,111.23 \implies 1+13s = 3.6111123$$

$$\implies s = \frac{3.6111123-1}{13} = 0.2008547923$$

$$i_{10} = \frac{a(10)-a(9)}{a(9)} = \frac{s}{1+(10-1)s} = \frac{0.2008547923}{1+(9)(0.2008547923)} = \boxed{0.071537}$$

Calculate the $d^{(4)}$ that is equivalent to an interest rate of 8% compounded monthly.

Solution:

$$\left(1+\frac{i^{(12)}}{12}\right)^{12}=\left(1-\frac{d^{(4)}}{4}\right)^{-4}$$

$$\left(1-\frac{d^{(4)}}{4}\right)^{-4}=\left(1+\frac{0.08}{12}\right)^{12} \implies \left(1-\frac{d^{(4)}}{4}\right)=\left(1+\frac{0.08}{12}\right)^{12/-4}$$

$$\implies \frac{d^{(4)}}{4}=1-\left(1+\frac{0.08}{12}\right)^{-3}$$

$$\implies d^{(4)}=4\left[1-\left(1+\frac{0.08}{12}\right)^{-3}\right]=\boxed{0.07895}$$

Deepa borrows 1000 from Ally. Deepa agrees to pay a nominal interest rate of 9% compounded quarterly.

At the end of X years, Deepa repays the loan with a payment of 1427.62.

Calculate X.

Solution:

$$i^{(4)} = 9\% \qquad \frac{i^{(4)}}{4} = \frac{9\%}{4} = 2.25\%$$

$$(1+i)^X = (1+\frac{i^{(4)}}{4})^{4X}$$

$$1000(1.0225)^{4X} = 1427.62 \quad ==> \quad (1.0225)^{4X} = 1.42762$$

$$4X = \frac{\ln(1.42762)}{\ln(1.0225)} = 16 \quad ==> \quad X = 4$$

You are given that $i = 8\%$.

Let $i^{(4)}$ be the nominal annual interest rate compounded quarterly that is equivalent to i.

Let $d^{(12)}$ be the nominal annual discount rate compounded monthly that is equivalent to i.

Calculate $(1000)(i^{(4)} - d^{(12)})$.

Solution:

$$(1+i) = 1.08 = \left(1 + \frac{i^{(4)}}{4}\right)^4 \Rightarrow i^{(4)} = 0.077706188$$

$$(1+i) = 1.08 = \left(1 - \frac{d^{(12)}}{12}\right)^{-12} \Rightarrow d^{(12)} = 0.076714776$$

$$\begin{aligned} 1000(i^{(4)} - d^{(12)}) &= 1000(0.077706188 - 0.076714776) \\ &= 0.991412 \end{aligned}$$

Amber has a loan which will be repaid with a lump sum at the end of five years. The amount of the loan is 40,000 and has an interest rate of 12% compounded quarterly.

Calculate the amount of interest that Amber will pay on this loan.

Solution:

After five years, Amber will have:

$$(40,000)\left(1+\frac{0.12}{4}\right)^{(4)(5)} = 72,244.45$$

The amount of interest will be $72,244.45 - 40,000.00 = 32,244.45$

Varun invests 10,000 in an account today.

During the first 5 years, the account earns an interest rate of 6% compounded quarterly.

During the second 5 years, the account earns a rate equivalent to a discount rate of 12% convertible monthly.

Determine the amount that Varun has at the end of 10 years.

Solution:

$$10,000\left(1+\frac{0.06}{4}\right)^{4(5)}\left(1-\frac{0.12}{12}\right)^{-12(5)}=10,000(1.015)^{20}(0.99)^{-60}$$
$$=24,615.53$$