Topic: Non-Level Annuities, Formula that does not follow the rules

Kelley is the beneficiary of the Wei Trust. The Wei Trust will pay Kelley quarterly payments for the next 10 years. The payments will be at the end of each quarter and will not be level.

The quarterly payments in the first year will each be 1000. The quarterly payments in the second year will each be 2000. The quarterly payments in the third year will each be 3000. The payments will continue to increase in the same pattern until quarterly payments of 10,000 will be made in the 10th year.

Using an annual effective interest rate of 8%, calculate the present value of Kelley's payments.

Solution:

To solve this problem, we note that the payments are level during each year but increase year to year, so we need to use the formula that does not follow the rules.

We are given $i = 0.08$ and we will need $\frac{i^{(4)}}{4}$

$$(1+i) = \left(1+\frac{i^{(4)}}{4}\right)^4 \Rightarrow \frac{i^{(4)}}{4} = (1.08)^{\frac{1}{4}} - 1 = 0.019426547$$

$$PV = (1000)\left(\frac{\left[\frac{1-(1.08)^{-10}}{0.08}\right](1.08) - 10(1.08)^{-10}}{0.019426547}\right) = 134,607.20$$

Brittney is receiving an annuity due with quarterly payments for 17 years. Each quarterly payment is 500 in the first year. Each quarterly payment is 1000 in the second year. Each quarterly payment is 1500 in the third year. The payments continue to increase in the same pattern until each quarterly payment is 8500 in the 17th year.

Using an interest rate of 8% compounded quarterly, calculate the present value of Brittney's annuity.

Solution:

First, we note that the payments are level during each year but increase year to year. Therefore, we know that it is the Formula That Does Not Follow The Rules. Secondly, we note that it is an annuity due.

$$i^{(4)} = 0.08 \Rightarrow \frac{i^{(4)}}{4} = \frac{0.08}{4} = 0.02$$

$$i = \left(1 + \frac{i^{(4)}}{4}\right)^4 - 1 = (1.02)^4 - 1 = 0.08243216$$

$$PV = (500)\left(\frac{\ddot{a}_{\overline{17}|} - 17(1+i)^{-17}}{\frac{i^{(4)}}{4}}\right)\left(1 + \frac{i^{(4)}}{4}\right)$$

$$= (500)\left(\frac{\left[\frac{1-(1.08243216)^{-17}}{0.08243216}\right](1.08243216) - 17(1.08243216)^{-17}}{0.02}\right)(1.02)$$

$$= 134,976.60$$

*We multiply the present value by (1.02) since it is an annuity due.

Owen invests quarterly deposits for 5 years with the Fairfield Fund. The deposits are made at the beginning of each quarter. The deposits during the first year are 10,000. The deposits during the second year are 20,000. The deposits during the third year are 30,000. The payments continue in the same pattern with payments increasing by 10,000 each year.

Fairfield Fund pays an annual effective interest rate of 7.5%.

Determine how much Owen will have in the Fairfield Fund at the end of 5 years.

Solution:

First we note that the payments are level during each year but increase year to year. Therefore, we know that it is the Formula That Does Not Follow The Rules. Secondly, we note that it is an annuity due. Finally, we note that we need the accumulated value.

We are given $i = 0.075$ and we will need $\dfrac{i^{(4)}}{4}$.

$$(1+i) = \left(1+\frac{i^{(4)}}{4}\right)^4 \implies \frac{i^{(4)}}{4} = (1.075)^{1/4} - 1 = 0.018244601$$

$$AV = (10,000)\left(\frac{\left[\dfrac{1-(1.075)^{-5}}{0.075}\right](1.075) - 5(1.075)^{-5}}{0.018244601}\right)(1.018244601)(1.075)^5 = 694,296.90$$

Jake makes deposits at the start of each quarter for ten years into the Rahn Fund. The Rahn Fund pays an annual effective interest rate of 6%.

The payments are P at the beginning of each quarter in the first year. The payments are $2P$ at the beginning of each quarter in the second year. The payments continue to increase each year until payments of $10P$ are paid at the beginning of each quarter in the tenth year.

At the end of 10 years, Jake has 100,000.

Determine P .

Solution:

We start by noting that this is an annuity due and that we need the accumulated value. We also note that since the payments are level for each year, we need to use the formula that does not follow the rules.

$$1+i=\left(1+\frac{i^{(4)}}{4}\right)^{4} \Longrightarrow 1.06=\left(1+\frac{i^{(4)}}{4}\right)^{4} \Longrightarrow \frac{i^{(4)}}{4}=(1.06)^{0.25}-1=0.014673846$$

$$P\left[\frac{\ddot{a}_{\overline{10}|0.06}-10(1.06)^{-10}}{0.014673846}\right](1.014673846)(1.06)^{10}=100,000$$

$$\ddot{a}_{\overline{10}|0.06}=\frac{1-(1.06)^{-10}}{0.06}(1.06)=7.801692273$$

$$P=\frac{100,000}{\left[\frac{7.801692273-10(1.06)^{-10}}{0.014673846}\right](1.014673846)(1.06)^{10}}=364.03$$

Ram buys an annuity immediate for his Mom. The annuity will make quarterly payments to his Mom for 20 years. The payments are 1000 each quarter in the first year. The payments are 1100 each quarter of the second year. The payments continue to increase in the same pattern until payments of 2900 are paid each quarter of the 20th year.

Using an interest rate of 8% compounded quarterly, calculate the price that Ram paid for this annuity. (The price is the present value of the payments.)

Solution:

To solve this problem, we need to use the formula that does not follow the rules. However, since the first payment does not equal the amount of the increase, we must split the annuity into level payments of 900 and payments that are 100 the first year, 200 the second year, etc.

To use the Formula that does not follow the rules, we need both $\frac{i^{(4)}}{4}$ and i.

$$\frac{i^{(4)}}{4} = \frac{0.08}{4} = 0.02 \qquad i = \left(1 + \frac{i^{(4)}}{4}\right)^4 - 1 = (1.02)^4 - 1 = 0.08243216$$

$$PV = 900 a_{\overline{80}|0.02} + 100\left[\frac{\ddot{a}_{\overline{20}|0.08243216} - 20(1.08243216)^{-20}}{0.02}\right]$$

$$= 900\left[\frac{1-(1.02)^{-80}}{0.02}\right] + 100\left[\frac{\frac{1-(1.08243216)^{-20}}{0.08243216}(1.08243216) - 20(1.08243216)^{-20}}{0.02}\right]$$

$$= 67,448.36$$

Kevin is receiving an annuity due with quarterly payments for the next 30 years. Each payment in the first year is P. Each payment in the second year is $2P$. Each payment in the third year is $3P$. The payments continue to increase by P each year until each payment in the 30th year is $30P$.

The present value of this annuity at an interest rate of 8% compounded quarterly is 200,000.

Determine P.

Solution:

Since this the formula that does not follow the rules, we need both $\frac{i^{(4)}}{4}$ and i.

We are given that $i^{(4)} = 0.08 \implies \frac{i^{(4)}}{4} = 0.02$ and $i = (1.02)^4 - 1 = 0.08243216$.

$$200,000 = P\left[\frac{\ddot{a}_{\overline{30}|0.08243216} - 30(1.08243216)^{-30}}{0.02}\right](1.02) =$$

$$P\left[\frac{\left(\frac{1-(1.08243216)^{-30}}{0.08243216}\right)(1.08243216) - 30(1.08243216)^{-30}}{0.02}\right](1.02) =$$

$$P\left[\frac{11.91140233 - 2.786866898}{0.02}\right](1.02) = P(465.356407)$$

$$P = \frac{200,000}{465.356407} = 429.78$$

The multiplication by (1.02) at the end of the formula is because this is "an annuity due".

Kevin invests in the White Fund. He makes deposits at the end of each month for 15 years. During the first year, each monthly deposit is 1000. During second year, each monthly deposit is 1100. During the third year, each monthly deposit is 1200. The deposits continue in the same pattern until each monthly deposits during the 15th year is 2400.

The White Fund pays an interest rate of 12% compounded monthly.

Calculate the amount that Kevin will have at the end of 15 years.

Solution:

For this problem, we must split the payments into level payments and payments that allow us to use the formula that does not follow the rules. The level payments will be the amount of the first payment less the amount of the increase $=1000-100=900$.

We need both i and $\frac{i^{(12)}}{12}$. We are given $i^{(12)}=0.12 ==> \frac{i^{(12)}}{12}=0.01$ and $i=(1.01)^{12}-1=0.12682503$

$$AV = PV(1.12682503)^{15}$$

$$AV = \left[900a_{\overline{180}|0.01} + 100\left(\frac{\ddot{a}_{\overline{15}|0.12682503} - 15(1.12682503)^{-15}}{0.01}\right)\right](1.12682503)^{15}$$

$$= \left[900\left(\frac{1-(1.01)^{-180}}{0.01}\right) + 100\left(\frac{\left\{\frac{1-(1.12682503)^{-15}}{0.12682503}\right\}(1.12682503) - 15(1.12682503)^{-15}}{0.01}\right)\right](1.12682503)^{15}$$

$$= 743,492.78$$

Wendy is the beneficiary of an annuity due which makes monthly payments for 15 years. Each monthly payment in the first year are 1000. Each monthly payment in the second year is 2000. The payments continue to increase until each monthly payment in the 15th year is 15,000.

Calculate the present value of Wendy's annuity at an interest rate of 9% compounded monthly.

Solution:

We have to use the formula that does not follow the rules since the payments are level during each year but increase year to year. We also note that this is an annuity due.

$$PV = 1000\left(\frac{\ddot{a}_{\overline{15}|i} - 15(1+i)^{-15}}{\frac{i^{(12)}}{12}}\right)\left(1+\frac{i^{(12)}}{12}\right)$$

$$\boxed{\frac{i^{(12)}}{12} = \frac{0.09}{12} = 0.0075 \quad \text{and} \quad i = (1.0075)^{12} - 1 = 0.093806898}$$

$$PV = 1000\left(\frac{\left[\frac{1-(1.093806898)^{-15}}{0.093806898}(1.093806898)\right] - 15(1.093806898)^{-15}}{0.0075}\right)(1.0075)$$

$$= 633,233.59$$