Topic: Outstanding Loan Balances

Abishek borrows 25,000 to buy a car. Abishek will repay the loan with 60 monthly payments. The interest rate on the loan is 9% compounded monthly.

Abishek makes the first 30 payments on time. Additionally, at the end of the 15th month, he makes an additional payment of 1000 over and above the normal payment.

Determine the outstanding loan balance right after the 30th payment.

Solution:

The monthly effective interest rate is: $\frac{i^{(12)}}{12} = \frac{0.09}{12} = 0.0075$.

$$Pa_{\overline{60}|} = 25,000 = P\left(\frac{1-(1.0075)^{-60}}{0.0075}\right) \quad ==> \quad 25,000 = 48.17P \quad ==> \quad P = 518.96$$

$$OLB_{30} = \text{ Accumulated Value of Past Cash Flows}$$

$$OLB_{30} = 25,000(1.0075)^{30} - 518.96s_{\overline{30}|} - 1000(1.0075)^{15}$$

$$= 25,000(1.0075)^{30} - 518.96\left(\frac{(1.0075)^{30}-1}{0.0075}\right) - 1000(1.0075)^{15}$$

$$= 31,281.79 - 17,386.63 - 1,118.60$$

$$= 12,776.56$$

****Disregard this practice problem if we have not yet covered loans or annuities with non-level payments****

Yue has a loan of 50,000 which is being repaid with non-level payments over 15 years. The first payment is 4000 at the end of one year. The second payment is 8000 at the end of two years. The third payment of 5000 at the end of three years. The fourth payment of 9000 at the end of four years.

The annual effective interest rate on the loan is 5%.

Determine the outstanding loan balance right after the payment of 5000 at the end of three years.

Solutions:

$$OLB_3 = \text{Accumulated Value of Past Cash Flows}$$

$$= (50,000)(1.05)^3 - 4000(1.05)^2 - 8000(1.05)^1 - 5000 = 40,071.25$$

****Disregard this practice problem if we have not yet covered loans or annuities with non-level payments****

Connor has borrowed money from Brow Bank. The loan has an annual effective interest rate of 5.25%.

Connor has agreed to repay the loan with 10 annual payments. The first payment is 100,000. The second payment is 90,000. Each payment is 10,000 less than the prior payment until a payment of 10,000 is paid at the end of the 10 year.

Determine the outstanding loan balance on this loan right after the seventh payment.

Solution:

$$OLB_7 = \text{Present Value of Future Payments}$$

$$= 30{,}000(1.0525)^{-1} + 20{,}000(1.0525)^{-2} + 10{,}000(1.0525)^{-3}$$

$$= 55{,}135.04$$

Adam is repaying a loan with monthly payments of 200 for 63 months. The interest rate on the loan is 9% compounded monthly.

Determine the outstanding loan balance of this loan at the end of one year which is twelve months.

Solution:

$$i^{(12)} = 0.09$$

$$\frac{i^{(12)}}{12} = 0.0075$$

$$OLB_{12} = 200a_{\overline{63-12}|} = 200\left(\frac{1-(1.0075)^{-51}}{0.0075}\right) = 8449.92$$

****Disregard this practice problem if we have not yet covered loans or annuities with non-level payments****

Maddie is repaying a loan with 10 payments. The payment at time t is $10,000t$. In other words, the first payment is 10,000. The second payment is 20,000. The payments continue to increase until a payment of 100,000 is made in the 10^{th} year.

The annual effective interest rate on the loan is 5%.

Maddie makes all the payments as scheduled except that she forgets to make the payment of 50,000 in the fifth year.

Determine the outstanding loan balance on Maddie's loan right after the payment of 80,000 is made in the 8^{th} year.

Solution:

The Outstanding Loan Balance at time 8 is the unpaid payments valued at time 8

$$OLB = (50,000)(1.05)^{3} + (90,000)(1.05)^{-1} + (100,000)(1.05)^{-2} = 234,298.48$$

****Disregard this practice problem if we have not yet covered loans or annuities with non-level payments****

Qian has a loan of 250,000 which has an annual effective interest rate of 5%. The loan will be repaid with annual payments for 15 years.

The first payment is 10,000. The second payment is 20,000. The third payment is 30,000. The fourth payment is 40,000. The fifth payment is 50,000. The payments at the end of the sixth through the 15th year are a level amount of Q.

Determine the outstanding loan balance right after the 4th payment.

Solution:

The easy way to do this is the retrospective approach which is the accumulated value of past payments.

$$OLB_4 = (250,000)(1.05)^4 - (10,000)(1.05)^3 - (20,000)(1.05)^2 - (30,000)(1.05) - 40,000$$

$$= 198,750.31$$

Disregard this practice problem if we have not yet covered loans or annuities with non-level payments

Qian has a loan of 250,000 which has an annual effective interest rate of 5%. The loan will be repaid with annual payments for 15 years.

The first payment is 10,000. The second payment is 20,000. The third payment is 30,000. The fourth payment is 40,000. The fifth payment is 50,000. The payments at the end of the sixth through the 15th year are a level amount of Q.

Determine Q.

Solution:

There are numerous methods to do this. I will do it multiple ways since this was the hardest question on the test.

One method is to use our point in time as time 0. Then the loan must equal the present value of the payments.

$$250,000 = (10,000)(1.05)^{-1} + (20,000)(1.05)^{-2} + (30,000)(1.05)^{-3} + (40,000)(1.05)^{-4} + (50,000)(1.05)^{-5} + Q(1.05)^{-5} a_{\overline{10|}}$$

$$= 125,663.93 + Q(1.05)^{-5}\left(\frac{1-(1.05)^{-10}}{0.05}\right) ==> Q = \frac{250,000 - 125,663.93}{(1.05)^{-5}\left(\frac{1-(1.05)^{-10}}{0.05}\right)} = 20,550.80$$

Another method is to set the OLB_5 based on retrospective calculation must be equal to OLB_5 based on prospective calculation.

Retrospectively

$$OLB_5 = (250,000)(1.05)^5 - (10,000)(1.05)^4 - (20,000)(1.05)^3 - (30,000)(1.05)^2 - 40,000(1.05) - 50,000$$

$$= 158,687.83$$

$$or\ OLB_5 = \left(OLB_4\right)(1.05) - 50,000 = 158,687.83$$

Prospectively $OLB_5 = Qa_{\overline{10|}} ==> Q = \dfrac{158,687.83}{\left(\frac{1-(1.05)^{-10}}{0.05}\right)} = 20,550.80$

Adam buys a new car for 30,000. Abbott Bank offers Adam the choice of two loans. Both loans have an interest rate of 12% compounded monthly.

Loan #1 is a five year loan with 60 monthly payments. This is a standard loan with the first payment at the end of the first month. The payment on this loan is Q.

Loan #2 is a five and one half year loan with 60 monthly payments. Under this loan, the payments are deferred for six months which means that the first payment will be made at the end of the 7^{th} month. The payment on this loan is X .

Determine $X - Q$.

Solution:

Payments are made monthly so we need the monthly effective interest rate of $\frac{i^{(12)}}{12}$. We are given that $i^{(12)} = 0.12 \Longrightarrow \frac{i^{(12)}}{12} = 0.01$

Loan #1

$$Qa_{\overline{60}|} = 30,000 \Longrightarrow Q = \frac{30,000}{\left(\frac{1-(1.01)^{-60}}{0.01}\right)} = 667.33$$

Or using your calculator $\Longrightarrow$ $\boxed{I/Y} \leftarrow 1; \boxed{PV} \leftarrow 30,000; \boxed{N} \leftarrow 60; \boxed{CPT}\boxed{PMT} \rightarrow 667.33$

Loan #2

$$Xv^6 a_{\overline{60}|} = 30,000 \Longrightarrow X = \frac{30,000}{(1.01)^{-6}\left(\frac{1-(1.01)^{-60}}{0.01}\right)} = 708.39$$

$$X - Q = 708.39 - 667.33 = 41.06$$