Topic: Perpetuities

A perpetuity due has a present value of 25,000 and makes quarterly payment of 500 per quarter.

Calculate the annual effective interest rate used to calculate the present value.

Solution:

$$(500)\left(\frac{1}{i}\right)(1+i) = 25,000 \Longrightarrow \frac{500}{i} + 500 = 25,000$$

$$\frac{500}{i} = 24,500 \Longrightarrow i = \frac{500}{24,500}$$

But since payments are quarterly, the above rate is a quarterly effective rate. We need the annual effective rate.

$$(1+i) = \left(1+\frac{i^{(4)}}{4}\right)^4 \Longrightarrow 1+i = \left(1+\frac{500}{24,500}\right)^4 \Longrightarrow i = 0.84165785$$

Mindy has won the lottery! She has the option of the following two payouts:

a. A perpetuity due with monthly payments of 50,000.

b. An annuity due for 35 years with annual payments of P.

The present value of the payments under either option calculated using an annual effective interest rate of i is 10 million which is 10,000,000.

Determine the payment P under Option b.

Solution:

From Option a, we can get the interest rate:

$$10,000,000 = (50,000)\left(\frac{1}{i}+1\right) ==> i = \frac{50,000}{9,950,000} = 0.005025126$$

This is the mothhly effective rate since payments are monthly. For Option b, we need the annual effective rate since the payments are annual. The annual effective interest rate is:

$$i = (1.005025126)^{12} - 1 = 0.061996372$$

$$PV = P\ddot{a}_{\overline{35}|} ==> 10,000,000 = P\left(\frac{1-(1.061996372)^{-35}}{0.061996372}\right)(1.061996372) ==> P = 664,746.82$$