Topic: Perpetuities

A perpetuity due has a present value of 25,000 and makes quarterly payment of 500 per quarter.

Calculate the annual effective interest rate used to calculate the present value.

Solution:

$$(500)\left(\frac{1}{i}\right)(1+i) = 25,000 \Longrightarrow \frac{500}{i} + 500 = 25,000$$

$$\frac{500}{i} = 24,500 \Longrightarrow i = \frac{500}{24,500}$$

But since payments are quarterly, the above rate is a quarterly effective rate. We need the annual effective rate.

$$(1+i) = \left(1 + \frac{i^{(4)}}{4}\right)^4 = > 1 + i = \left(1 + \frac{500}{24,500}\right)^4 = > i = 0.084165785$$

Mindy has won the lottery! She has the option of the following two payouts:

- a. A perpetuity due with monthly payments of 50,000.
- b. An annuity due for 35 years with annual payments of P.

The present value of the payments under either option calculated using an annual effective interest rate of i is 10 million which is 10,000,000.

Determine the payment P under Option b.

Solution:

From Option a, we can get the interest rate:

$$10,000,000 = (50,000) \left(\frac{1}{i} + 1\right) = > i = \frac{50,000}{9,950,000} = 0.005025126$$

This is the mothhly effective rate since payments are monthly. For Option b, we need the annual effective rate since the payments are annual. The annual effective interest rate is:

$$i = (1.005025126)^{12} - 1 = 0.061996372$$

$$PV = P\ddot{a}_{\overline{35}} = > 10,000,000 = P\left(\frac{1 - (1.061996372)^{-35}}{0.061996372}\right) (1.061996372) = > P = 664,746.82$$