

### Topic: Time-Weighted Yield

On January 1, 2016, Natalie invested 100,000 in an account. On May 31, 2016, Natalie's account was worth 110,000 and she invested an additional 45,000. On August 1, 2016, Natalie's account was worth 150,000. At that time, Natalie withdrew 25,000 to pay her tuition. On December 31, 2017, Natalie's account was now worth 135,000 and Natalie decided to withdraw all her money.

Calculate Natalie's annual time weighted return for this account.

**Solution:**

$$1 + j_1 = \frac{110,000}{100,000}$$

$$1 + j_2 = \frac{150,000}{155,000}$$

$$1 + j_3 = \frac{135,000}{125,000}$$

$$1 + j_{tw} = (1 + j_1)(1 + j_2)(1 + j_3) = \left(\frac{110,000}{100,000}\right)\left(\frac{150,000}{155,000}\right)\left(\frac{135,000}{125,000}\right) = 1.1496774$$

$$1 + i_{TW} = (1 + j_{tw})^{1/2} = 1.0722301$$

$$i_{TW} = 0.072230$$

1. Alex has 100,000 invested in the Hahn Investment Fund today, September 19, 2017.

On January 19, 2018, Alex has a balance of 105,000 and he withdraws 12,000 to pay his tuition for next semester.

On May 19, 2018, Alex has a balance of 90,000. He withdraws 10,000 to buy a new car.

At the end of the summer on August 19, 2018, Alex deposits 30,000 which is his earnings from his summer internship. Prior to making that deposit, Alex had 85,000 in his account.

On September 19, 2019, Alex has a balance of 122,000 in the Hahn Investment Fund.

Determine Alex's annual time weighted return during his two year investment in the Hahn Investment Fund.

**Solution:**

$$1 + j_{TW} = (1 + j_1)(1 + j_2)(1 + j_3)(1 + j_4)$$

$$1 + j_1 = \frac{\text{Amount at end of period before cash flow}}{\text{Amount at start of period after any cash flow}} = \frac{105,000}{100,000}$$

$$1 + j_2 = \frac{90,000}{105,000 - 12,000} = \frac{90,000}{93,000}$$

$$1 + j_3 = \frac{85,000}{90,000 - 10,000} = \frac{85,000}{80,000}$$

$$1 + j_4 = \frac{122,000}{85,000 + 30,000} = \frac{122,000}{115,000}$$

$$1 + j_{TW} = (1 + j_1)(1 + j_2)(1 + j_3)(1 + j_4) = \left(\frac{105}{100}\right)\left(\frac{90}{93}\right)\left(\frac{85}{80}\right)\left(\frac{122}{115}\right) = 1.145354137$$

$$1 + i_{TW} = (1 + j_{TW})^{\frac{1}{T}} = (1.145354137)^{\frac{1}{2}} \implies i_{TW} = 0.070212$$