**Chapter 2 – Past Test and Quiz Problems – Survival Function, p & q values, Force of Mortality**

(3 points)

You are given that $_t p_{90} = 1 - 0.04t^2$ for $0 \le t \le 5$.

Calculate $\mu_{91}$.

**Solution:**

$$\mu_{90+t} = -\frac{\frac{d}{dt}\,_t p_{90}}{_t p_{90}} = \frac{0.08t}{1 - 0.04t^2}$$

$$\mu_{91} = \frac{0.08(1)}{1 - 0.04(1)^2} = \boxed{0.0833333}$$

(3 points)

You are given that $\mu_{80+t} = 0.04t$ .

Calculate $_{10}P_{80}$ .

**Solution:**

$$_{10}P_{80} = e^{-\int_0^{10} \mu_{80+t} \cdot dt} = e^{-\int_0^{10} 0.04t \cdot dt} = e^{-\left[0.02t^2\right]_0^{10}} = e^{-2} = \boxed{0.13534}$$

(8 points)

You are given that $\mu_x = 0.001x + 0.01$ .

Calculate $_{10}q_{50}$ .

**Solution:**

$$S_0(x) = e^{-\int_0^x \mu_r \, dr} = e^{-\int_0^x (0.001r + 0.01) \, dr} = e^{-[0.0005r^2 + 0.01r]_0^x} = e^{-0.0005x^2 - 0.01x}$$

$$S_x(t) = \frac{S_0(x+t)}{S_0(x)} = \frac{e^{-0.0005(x+t)^2 - 0.01(x+t)}}{e^{-0.0005(x)^2 - 0.01(x)}}$$

$$_{10}q_{50} = 1 - {}_{10}p_{50}$$

$$_t p_x = \frac{e^{-0.0005(x+t)^2 - 0.01(x+t)}}{e^{-0.0005(x)^2 - 0.01(x)}}$$

$$_{10}p_{50} = \frac{e^{-0.0005(60)^2 - 0.01(60)}}{e^{-0.0005(50)^2 - 0.01(50)}} = \frac{0.090718}{0.173774} = 0.52205$$

$$_{10}q_{50} = 1 - 0.52205 = 0.47795$$

(6 points)

You are given that $S_0(x) = 1 - \dfrac{x^2}{6400}$ for $0 \le x \le 80$ .

If $\mu_x = \dfrac{17}{222}$ , determine $x$.

**Solution:**

$$\mu_x = \dfrac{-\dfrac{d}{dx} S_0(x)}{S_0(x)}$$

$$\mu_x = \frac{17}{222} = \frac{-\dfrac{d}{dx}\left(1 - \dfrac{x^2}{6400}\right)}{1 - \dfrac{x^2}{6400}} \Longrightarrow \frac{17}{222} = \frac{2x}{6400 - x^2} \Longrightarrow (17)(6400 - x^2) = (222)(2x)$$

$$\therefore 17x^2 + 444x - 108,800 = 0$$

$$x = \frac{-444 \pm \sqrt{444^2 - 4(17)(-108,000)}}{2(17)} = \frac{-444 \pm 2756}{34} \rightarrow x = 68, -94.12$$

$$x = 68$$