Chapter 3 – Past Test and Quiz Problems – l values, Constant Force of Mortality

(6 points) You are given the following mortality table:

x	q_x
90	0.1
91	0.3
92	0.5
93	0.7
94	1.0

For ages 90 to 91 and 91 to 92, deaths are uniformly distributed between integral ages.

For ages 92 to 93 and 93 to 94, there is a constant force of mortality between integral ages.

Calculate ${}_{0.8|1.5}q_{90.6}$.

Solution:

$${}_{0.8|1.5}q_{90.6} = \frac{l_{91.4} - l_{92.9}}{l_{90.6}}$$

$$l_{90} = 1000; l_{91} = (1000)(1-0.1) = 900; l_{92} = (900)(1-0.3) = 630; l_{93} = (630)(1-0.5) = 315$$

$$l_{90.6} = (1-0.6)(1000) + (0.6)(900) = 940$$
$$l_{91.4} = (1-0.4)(900) + (0.4)(630) = 792$$
$$l_{92.9} = (630)^{(1-0.9)}(315)^{0.9} = 337.60864$$

$${}_{0.8|1.5}q_{90.6} = \frac{l_{91.4} - l_{92.9}}{l_{90.6}} = \frac{792 - 337.60864}{940} = 0.4834$$

(5 points) You are given the following mortality table:

x	q_x
90	0.2
91	0.4

You are also given that deaths are uniformly distributed between ages 90 and 91 and the deaths follow a constant force of mortality between ages 91 and 92.

Calculate ${}_{0.2|0.9}q_{90.2}$.

Solution:

$$l_{90} = 1000$$
$$l_{91} = 1000(1-0.2) = 800$$
$$l_{92} = 800(1-0.4) = 480$$

$${}_{0.2|0.9}q_{90.2} = \frac{l_{90.4} - l_{91.3}}{l_{90.2}}$$

$$= \frac{[1000(0.6) + 800(0.4)] - 800^{0.7}(480)^{0.3}}{1000(0.8) + 800(0.2)}$$

$$= \frac{233.6662}{960} = 0.2434$$

(6 points) You are given that Mortality follows the Standard Ultimate Life Table.

You are also given that mortality between 90 and 91 and between 91 and 92 follows a constant force of mortality while deaths are uniformly distributed between 92 and 93 and between 93 and 94.

Calculate ${}_{1.6\,|\,1.2}q_{90.8}$.

Solution:

$${}_{1.6\,|\,1.2}q_{90.8} = \frac{l_{92.4} - l_{93.6}}{l_{90.8}}$$

$$l_{90.8} = (l_{90})^{0.2}(l_{91})^{0.8} = (41,841.1)^{0.2}(37,618.6)^{0.8} = 38,427.55$$

$$l_{92.4} = (0.6)(33,379.9) + (0.4)(29,183.8) = 31,701.46$$

$$l_{93.6} = (0.4)(29,183.8) + (0.6)(25,094.3) = 26,730.1$$

$${}_{1.6\,|\,1.2}q_{90.8} = \frac{31,701.46 - 26,730.1}{38,427.55} = 0.12937$$

(6 points) You are given the following values from a mortality table:

$$q_{90} = 0.25 \qquad q_{91} = 0.40 \qquad q_{92} = 0.50$$

You are also given that deaths are uniformly distributed between 90 and 91 and between 91 and 92. Mortality between 92 and 93 follows a constant force of mortality.

Calculate ${}_{0.7\,|\,1.3}q_{90.6}$.

Solution:

$${}_{0.7\,|\,1.3}q_{90.6} = \frac{l_{91.3} - l_{92.6}}{l_{90.6}}$$

$$l_{90} = 1000 ==> l_{91} = (1000)(1-0.25) = 750 ==> l_{92} = 750(1-0.4) = 450 ==> l_{93} = (450)(1-0.5) = 225$$

$$l_{90.6} = (0.4)(1000) + (0.6)(750) = 850$$

$$l_{91.3} = (0.7)(750) + (0.3)(450) = 660$$

$$l_{92.6} = (450)^{0.4}(225)^{0.6} = 296.88928$$

$${}_{0.7\,|\,1.3}q_{90.6} = \frac{660 - 296.8828}{850} = 0.42719$$

(6 points) You are given:

a. $q_{85} = 0.2$

b. $q_{86} = 0.4$

c. Deaths are uniformly distributed between age 85 and 86.

d. There is a constant force of mortality between age 86 and 87.

Calculate ${}_{0.4|0.7}q_{85.6}$.

Solution:

$${}_{0.4|0.7}q_{85.6} = \frac{l_{86} - l_{86.7}}{l_{85.6}} = \frac{800 - 559.49455}{880} = 0.27330$$

$$l_{85} = 1000$$

$$l_{86} = (1000)(1 - 0.2) = 800$$

$$l_{87} = 800(1 - 0.4) = 480$$

$$l_{85.6} = (1000)(1 - 0.6) + 800(0.6) = 880 \quad \text{since it is UDD}$$

$$l_{86.7} = (800)^{(1-0.7)} \cdot (480)^{0.3} = 559.49445$$